Lecture 2
Review of
Signals and Systems:
Part 1
Block Diagrams of Communication System

**Digital Communication System**

- **Information (sound, video, text, data, ...)**
- **Transducer & A/D Converter**
- **Source Encoder**
- **Channel Encoder**
- **Modulator**
- **Tx RF System**
- **Channel**
- **Demodulator**
- **Rx RF System**
- **D/A Converter and/or output transducer**
- **Source Decoder**
- **Channel Decoder**
- **Output Signal**

- □ = Discrete-Time
- □ = Continuous-Time

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EE4900/EE6720 Digital Communications
Why Signals and Systems?

- Wireless/Wired Communications is fundamentally a *transmission-of-information* problem
- Information is transmitted through channel/medium
- Information is transmitted via *signals* through *systems*
- Real-world (outside of the embedded hardware) consists of phenomenon of *continuous-time, t*
- *After sampling*, the embedded hardware can be thought to run on *discrete-time, nT*

![Diagram of signal processing](image)
Continuous-time Signals

- Energy and Power of periodic signal, $x(t)$
  
  Energy: eq. 2.2
  Power: eq. 2.6

- Autocorrelation function, $r_x(t)$ for energy signal, eq. 2.4

- Autocorrelation function, $\phi_x(t)$ for power signal, eq. 2.5

- Fourier-series representation of the signal $x(t)$, eq.2.7-2.9
Continuous-time Signals

- Fourier-series representation of the signal, $x(t)$

A periodic signal can be represented by its Fourier series as an infinite sum of sines and cosines
Continuous-time Signals

- **The Impulse Function, \( \delta(t) \)**
- The Impulse function is zero everywhere except at \( t=0 \): eq. 2.10
- The sifting property (sampling):

\[
\int_{t_1}^{t_2} x(t) \delta(t - \tau) \, dt = \begin{cases} 
x(\tau), & t_1 \leq \tau \leq t_2 \\
0, & \text{elsewhere}
\end{cases}
\]
The Unit-step Function, $u(t)$

- The Unit-step function is 1 everywhere except $t \leq 0$
- The Unit-step function represents switching (on/off):

$$u(t) = \begin{cases} 
0, & t < 0 \\
\text{undefined}, & t = 0 \\
1, & t > 0 
\end{cases}$$
Discrete-time Signals

- When a continuous-time signal $x(t)$ is sampled, it becomes a discrete-time signal $x(n)$

- Energy and Power of periodic signal, $x(n)$
  
  Energy: eq. 2.13
  
  Power: eq. 2.14

- Fourier-series representation of the signal $x(n)$, eq. 2.15-2.17

- Discrete-time impulse function $\delta(n)$ and step function $u(n)$

$$\delta(n) = \begin{cases} 
1, & n = 0 \\
0, & n \neq 0
\end{cases}$$

$$u(n) = \begin{cases} 
1, & n \geq 0 \\
0, & n < 0
\end{cases}$$
Continuous-time Systems

- Continuous-time system is a collection of continuous-time components such as resistors, capacitors, inductors, transistors
- Linear, Time-Invariant (LTI) System produces output $y(t)$ with \textbf{same delay} as the input $x(t)$
- LTI System is characterized by its impulse response $h(t)$

**Convolution Integral**: find the output $y(t)$ given the input $x(t)$ for the system with impulse response $h(t)$
Continuous-time Systems

- Convolution: eq. 2.21
- Application of Convolution: Filter
  - When two signals are convolved in time-domain, they can be multiplied in frequency-domain: filter

![Diagram of convolution process]

**Time-Domain**

\[ x(t) \quad h(t) \quad y(t) = x(t) * h(t) \]

**Frequency-Domain**

\[ X(\omega) \quad H(\omega) \quad Y(\omega) = X(\omega)H(\omega) \]
Continuous-time Systems

- Laplace Transform, eq.2.26-2.27
- Complex Frequency, $s=\sigma+j\omega$
- Purpose: convert time-domain signal into frequency-domain (s-domain) signal

<table>
<thead>
<tr>
<th>$x(t)$</th>
<th>$X(s)$</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(t)$</td>
<td>1</td>
<td>all $s$</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>$\frac{1}{s}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
<tr>
<td>$t^{n-1}u(t)$</td>
<td>$\frac{1}{s^n}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
<tr>
<td>$e^{-at}u(t)$</td>
<td>$\frac{1}{s+a}$</td>
<td>Real ${s} &gt; -a$</td>
</tr>
<tr>
<td>$t^{n-1}e^{-at}u(t)$</td>
<td>$\frac{1}{(s+a)^n}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
<tr>
<td>$\delta(t-T)$</td>
<td>$e^{-sT}$</td>
<td>all $s$</td>
</tr>
<tr>
<td>$\cos(\omega_0 t)u(t)$</td>
<td>$\frac{s}{s^2 + \omega_0^2}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
<tr>
<td>$\sin(\omega_0 t)u(t)$</td>
<td>$\frac{\omega_0}{s^2 + \omega_0^2}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
<tr>
<td>$e^{-at}\cos(\omega_0 t)u(t)$</td>
<td>$\frac{s}{(s+a)^2 + \omega_0^2}$</td>
<td>Real ${s} &gt; -a$</td>
</tr>
<tr>
<td>$e^{-at}\sin(\omega_0 t)u(t)$</td>
<td>$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$</td>
<td>Real ${s} &gt; -a$</td>
</tr>
<tr>
<td>$\frac{d^n\delta(t)}{dt^n}$</td>
<td>$\frac{s^n}{s^n}$</td>
<td>all $s$</td>
</tr>
<tr>
<td>$u(t) * \cdots * u(t)$</td>
<td>$\frac{1}{s^n}$</td>
<td>Real ${s} &gt; 0$</td>
</tr>
</tbody>
</table>
Continuous-time Systems

- Pole-zero plot: poles lift the amplitude in s-domain, zeros pin the amplitude in s-domain
Continuous-time Systems

- Fourier Transform: a special case of Laplace Transform, with $s = \sigma + j\omega$; where $\sigma = 0$. Recall that this indicates sustained oscillations (critically-damped case)

![Laplace Transform](image1)

![Fourier Transform](image2)
Continuous-time Systems

**Fourier Transform, eq.2.39-2.40**

<table>
<thead>
<tr>
<th>x(t)</th>
<th>X(jω)</th>
<th>X(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ(t)</td>
<td>1</td>
<td>e^{-j2πf_0}</td>
</tr>
<tr>
<td>δ(t - t_0)</td>
<td>1 / (jω + πδ(ω))</td>
<td>1 / (j2πf + πδ(f))</td>
</tr>
<tr>
<td>u(t)</td>
<td>1 / (a + jω)^n</td>
<td>1 / (a + j2πf)^n</td>
</tr>
<tr>
<td>t^n / (n-1)! e^{-at} u(t)</td>
<td>∑ a_k e^{jkω_0 t}</td>
<td>2π ∑ a_k δ(ω - kω_0)</td>
</tr>
<tr>
<td>∑ δ(t - kT)</td>
<td>2πT sin(ωT) / ωT</td>
<td>2T sin(2πfT) / 2πfT</td>
</tr>
<tr>
<td>f &lt; T</td>
<td>2πδ(ω - ω_0)</td>
<td>2πδ(ω - ω_0)</td>
</tr>
<tr>
<td>f &gt; T</td>
<td>0</td>
<td>δ(f - f_0)</td>
</tr>
<tr>
<td>e^{jω_0 t}</td>
<td>2πδ(ω - ω_0)</td>
<td>δ(f - f_0)</td>
</tr>
<tr>
<td>cos(ω_0 t)</td>
<td>πδ(ω - ω_0) + πδ(ω + ω_0)</td>
<td>1 / 2 δ(f - f_0) + 1 / 2 δ(f + f_0)</td>
</tr>
<tr>
<td>sin(ω_0 t)</td>
<td>jπδ(ω - ω_0) - jπδ(ω + ω_0)</td>
<td>1 / j2δ(f - f_0) - 1 / j2δ(f + f_0)</td>
</tr>
<tr>
<td>exp(-a</td>
<td>t</td>
<td>)</td>
</tr>
<tr>
<td>exp(-πt^2)</td>
<td>exp(-π(ω / 2π)^2)</td>
<td>exp(-πf^2)</td>
</tr>
</tbody>
</table>
Energy of Baseband signal vs. Bandpass signal

(a) Baseband signal

\[ |X(f)|^2 \]

-\( B \) \quad 0 \quad +B

(b) Bandpass signal

\[ |X(f)|^2 \]

-\( f_c - B \) \quad -f_c \quad -f_c + B \quad 0 \quad f_c - B \quad f_c \quad f_c + B
Examples of Discrete-time Systems: embedded computing with memory and microprocessor, digital logic circuits

Convolution Sum: find the output samples $y(n)$ given the input samples $x(n)$ for the system with impulse response $h(n)$

$$y(n) = x(n) * h(n)$$

$$X(\Omega) = H(\Omega)$$

Frequency-Domain

Time-Domain
Linear Constant-coefficient Difference Equation

- Input and output relationship of discrete-time system
- Implemented by multipliers, adders, delay (memory) blocks
- Suppose that
  1) Input signal samples are denoted by \(x(n)\) and output by \(y(n)\)
  2) The filter coefficients are denoted by numerator \(b_0, \ldots, b_M\) and denominator \(a_0, \ldots, a_M\) with filter order=\(M\)

Equation 2.24
Z-Transform, eq.2.45-2.46

Complex Frequency, \( z = \text{Re}\{z\} + j \text{Im}\{z\} \)

**Purpose:** convert
discrete time-domain signal into discrete frequency-domain (z-domain) signal

<table>
<thead>
<tr>
<th>Signal</th>
<th>Transform</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta(n) )</td>
<td>1</td>
<td>all ( z )</td>
</tr>
<tr>
<td>( u(n) )</td>
<td>( \frac{1}{1 - z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( a^n u(n) )</td>
<td>( \frac{1}{1 - az^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( na^n u(n) )</td>
<td>( \frac{az^{-1}}{(1 - az^{-1})^2} )</td>
<td>(</td>
</tr>
<tr>
<td>( \delta(n - m) )</td>
<td>( \frac{z^{-m}}{1} )</td>
<td>all ( z )</td>
</tr>
<tr>
<td>( \cos (\Omega_0 n) u(n) )</td>
<td>( \frac{1 - \cos (\Omega_0)z^{-1}}{1 - [2 \cos (\Omega_0)]z^{-1} + z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>( \sin (\Omega_0 n) u(n) )</td>
<td>( \frac{\sin (\Omega_0)z^{-1}}{1 - [2 \cos (\Omega_0)]z^{-1} + z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>( r^n \cos (\Omega_0 n) u(n) )</td>
<td>( \frac{1 - r \cos (\Omega_0)z^{-1}}{1 - [2r \cos (\Omega_0)]z^{-1} + z^{-2}} )</td>
<td>(</td>
</tr>
<tr>
<td>( r^n \sin (\Omega_0 n) u(n) )</td>
<td>( \frac{r \sin (\Omega_0)z^{-1}}{1 - [2r \cos (\Omega_0)]z^{-1} + z^{-2}} )</td>
<td>(</td>
</tr>
</tbody>
</table>

\(^t\) except 0 if \( m > 0 \) or \( \infty \) if \( m > 0 \)
Discrete-time Systems

- Pole-zero plot: poles lift the amplitude in $z$-domain, zeros pin the amplitude in $z$-domain
- Discrete Fourier Transform: a special case of $Z$-Transform, with $z = e^{j\Omega}$
### Discrete-time Fourier Transform (DTFT), eq. 2.54-2.55

#### Table 2.4.8 Some DTFT pairs

<table>
<thead>
<tr>
<th>Signal</th>
<th>DTFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(n)$</td>
<td>$X(e^{j\Omega})$</td>
</tr>
<tr>
<td>$\delta(n)$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\delta(n - n_0)$</td>
<td>$e^{-j\Omega n_0}$</td>
</tr>
<tr>
<td>$u(n)$</td>
<td>$\frac{1}{1 - e^{-j\Omega}} + \sum_{l=-\infty}^{\infty} \delta(\Omega - 2\pi l)$</td>
</tr>
<tr>
<td>$a^n u(n),</td>
<td>a</td>
</tr>
<tr>
<td>$\sum_{k=k_0}^{k_0+N-1} c_k e^{j(2\pi/N)n}$</td>
<td>$2\pi \sum_{k=-\infty}^{\infty} c_k \delta\left(\Omega - \frac{2\pi k}{N}\right)$</td>
</tr>
<tr>
<td>$\sum_{k=-\infty}^{\infty} \delta(n - kN)$</td>
<td>$\frac{2\pi}{N} \sum_{l=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi l}{N}\right)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2\pi \sum_{l=-\infty}^{\infty} \delta(\Omega - 2\pi l)$</td>
</tr>
<tr>
<td>$\left{ \begin{array}{ll} 1 &amp;</td>
<td>n</td>
</tr>
<tr>
<td>$e^{j\Omega_0 n}$</td>
<td>$2\pi \sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi l)$</td>
</tr>
<tr>
<td>$\cos(\Omega_0 n)$</td>
<td>$\pi \sum_{l=-\infty}^{\infty} \left[\delta(\Omega - \Omega_0 - 2\pi l) + \delta(\Omega + \Omega_0 - 2\pi l)\right]$</td>
</tr>
<tr>
<td>$\sin(\Omega_0 n)$</td>
<td>$\frac{\pi}{\Omega} \sum_{l=-\infty}^{\infty} \left[\delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)\right]$</td>
</tr>
</tbody>
</table>
Discrete-time Systems

- Periodicity of DTFT, \( X(e^{j\Omega}) \) is periodic at every \( 2\pi \)

1 period of the discrete-time signal

3 periods of the discrete-time signal

4 periods of the discrete-time signal
**Discrete-time Systems**

- $\Omega$ is a continuous variable! To represent the DTFT in digital logic hardware, $\Omega$ must be sampled (discrete values)
- When the DTFT $X(e^{j\Omega})$ is **sampled** at interval $2\pi/N$, it is called Discrete Fourier Transform (DFT): eq.2.57-2.58
**Sampling Theorem: Section 2.6.1**

**Application: A/D**

\[ x_c(t) \longrightarrow x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT) \delta(t-nT) \]

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) \]

Diagram showing the continuous-time signal \( x_c(t) \) and its sampled version \( x_p(t) \) in the time domain, along with their frequency domain representations \( P(j\omega) \) and \( X_p(j\omega) \).
Ideal Low Pass Filtering

**Application: D/A**

\[ x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT) \delta(t-nT) \]

\[ \rightarrow H(j\omega) \rightarrow x_c(t) \]

Discrete-time samples

LPF

Continuous-signal

\[ \begin{align*}
X_p(j\omega) & \quad \frac{1}{T} \\
H(j\omega) & \quad T \\
X_c(j\omega) & \quad \frac{2\pi}{T} - W
\end{align*} \]
Discrete-time Processing

Slide 23

\[ x_d(n) = x_c(nT) \]

ADC

discrete-time processing

DAC

\[ y_d(n) = y_c(nT) \]

\[ x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT) \delta(t - nT) \]

extract areas and convert to discrete-time sequence

\[ x_d(n) = x_c(nT) \]

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \]