Here are some hard problems from this semester

Intersection and Union:
Solve the inequality $|2x-1| > 7$. The solution is all values of x such that $2x-1 > 7$ or $2x-1 < -1$. To solve $2x-1 > 7$, just add 1 to both sides and divide by 2. We get $x > 4$. To solve $2x-1 < -1$, we do the same thing but get $x < 0$. Thus the solution is $(-\infty, 0) \cup (4, \infty)$. The reason it is a union is that we are looking for one condition **or** the other (as opposed to **and**).

Setting up a system of equations:
Traveling for 3 hr into a steady headwind, a plane flies 1650 mi. The pilot determines that flying with the same wind for 2 hr, he could make a trip of 1300 mi. Find the speed of the plane and the speed of the wind.
Solution: Let p be the speed of the plane in still air, and let w be the speed of the wind. Relative to the ground, the plane's speed is $p-w$ when going into the wind, the speed relative to the ground is $p+w$. We are basically told that $(p-w)(3) = 1650$ and $(p+w)(2) = 1300$. This gives us the system

$$3p - 3w = 1650$$
$$2p + 2w = 1300.$$

I trust you to solve this system.

Simplifying a nasty expression:
Simplify

$$\frac{\frac{x+3}{x} - \frac{4}{x-1}}{\frac{x}{x-1} + \frac{1}{x}}.$$

Answer: Multiply on both the top and bottom by $x(x-1)$. Then you have

$$\frac{\frac{x+3}{x} - \frac{4}{x-1}}{\frac{x}{x-1} + \frac{1}{x}} \frac{x(x-1)}{x(x-1)} = \frac{(x+3)(x-1) - 4x}{x^2 + (x-1)} = \frac{x^2 - 2x - 3}{x^2 + x - 1}.$$

Dividing one polynomial by another:
What is $(m^3 + \frac{7}{2}m + 3) \div (2m + 3)$? **Answer:** It is hard for me to typeset the solution, so I'll just state that the answer is $\frac{1}{2}m^2 - \frac{3}{4}m + \frac{23}{8}$ with a remainder of $\frac{-45}{8}$.

Sharing work: Ricky Bobby and Cal Naughton are refinishing a table. Working alone, Cal could do the job in 7 hr. If the two work together, the job takes 5 hr. How long will it take Ricky Bobby to refinish the table working alone?

Answer: Note that Cal can do 1/7 of the job per hour, and thus in 5 hours, he does (5)(1/7) of the whole job. If Ricky Bobby can do the job in x hours, then he can do $1/x$ of the job per hour and (therefore) $(5)(1/x)$ of the job in 5 hours. Since they do 1 whole job in 5 hours, it follows that

$$5(1/x) + 5(1/7) = 1,$$

or equivalently $5/x + 5/7 = 1$. This means that $x = 17.5$ hours. But I thought Ricky Bobby was fast!

Rates: An inlet pipe can fill an artificial lily pond in 60 min, while an outlet pipe can empty it in 80 min. Through an error, both pipes are left open. How long will it take for the pond to fill.
Answer: Let x be the number of minutes it takes to fill the pond in this manner. Water comes in at a rate of 1/60 of a pond per minute, and water leaves at a rate of 1/80 of a pond per minute. After x minutes, we have that the amount of water to enter is (x min) $\left(\frac{1}{60}\right.$ pond per minute$\left.\right) = x/60$ of the pond, and the amount of water to leave is (x min) $\left(\frac{1}{80}\right.$ pond per minute$\left.\right) = x/80$ of the pond. This means that the fraction of the pond is $x/60 - x/80$ full. We want to know when that fraction is 1 (whole pond). The answer is whenever

$$x/60 - x/80 = 1.$$

This gives us $80x - 60x = 4800$, and thus $x = \frac{4800}{20} = 2400$ minutes.