SMSG Postulates

1. (Two Points Determine a Line) Given any two different points, there is exactly one line that contains them both.
2. (Distance Postulate) To every pair of different points, there corresponds a unique positive number.

   For points, A and B, this unique positive number is denoted by $d(A, B)$, and is referred to as the distance between A and B.
3. (Ruler Postulate) The points of a line can be put into one-to-one correspondence with the real numbers in such a way that

   (i) to every point, there corresponds exactly one real number called the point's *coordinate*

   (ii) to every real number, there corresponds exactly one point of the line, and

   (iii) the distance between two points is the absolute value of the difference of the corresponding coordinates.
4. (Ruler Placement Postulate) Given two points P and Q of a line, the coordinate system (i.e. the one-to-one correspondence) can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

5-8. Postulates 5 through 8 deal with geometry of three dimensions, and we ignore them here.

9. (Plane Separation) Given a line ℓ and a plane containing it, the points of the plane α that do not lie on the line form two nonempty sets such that

   (i) each of the sets is convex, and

   (ii) if point P is in one set and point Q is in the other, then $\overline{PQ} \cap \ell \neq \emptyset$.
10. (Space Separation) We ignore this postulate for now (since it deals with three dimensions).
11. (Angle Measurement) To every angle $\angle ABC$, there corresponds a unique real number between 0 and 180, which we denote by $m\angle ABC$.
12. (Angle Construction) Let $\overrightarrow{AB}$ be a ray on the edge of half-plane H. For every number r between 0 and 180, there is exactly one ray $\overrightarrow{AP}$, with P in H such that $m\angle PAB = r$.
13. (Angle Addition) If D is a point in the interior of $\angle ABC$ and not on $\overrightarrow{BA}$ or $\overrightarrow{BC}$, then $m\angle ABD + m\angle DBC = m\angle ABC$.
14. (Supplement) If two angles form a linear pair, then they are supplementary.
15. (SAS Congruence for Triangles) Suppose we are given a correspondence of vertices and sides between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence of triangles.

Postulates (axioms) 1 through 15 above are the axioms for *neutral* (or *absolute*)geometry.

16. (Parallel Postulate) Given a line ℓ and a point P not on ℓ, there is at most one line through P that is parallel to ℓ. (Assuming this postulate makes our geometry *Euclidean*.)
17. (Area) To every polygonal region, there corresponds a unique positive number called the *area* of the region.
18. (Congruence versus Area) If two triangles are congruent, then the triangular regions have the same area.
19. (Additivity of Area) Suppose that the region R is the union of two regions R_1 and R_2. Suppose also that R_1 and R_2 intersect at most in a finite number of segments and points. Then the area of R is the sum of the areas of R_1 and R_2.
20. (Area of Rectangle) The area of a rectangle is equal to the product of the length of its base and the length of its altitude.
21. (Volume) Ignore this.
22. (Cavalieri's Principle) Ignore this.

SMSG Definitions

1. For distinct points A, B, and C, we say C *lies between* A and B, denoted $A - C - B$, if A, B, C are distinct and $d(A, B) = d(A, C) + d(C, B)$.
2. For two distinct points A, B a *line segment* $\overline{AB}$ consists of the points A, B and of all points C on the unique line passing through A and B such that $A - C - B$. The points A and B are called the endpoints of $\overline{AB}$.
3. (Notation) For two distinct points A, B, the unique line through A and B is denoted $\overleftrightarrow{AB}$.
4. For two distinct points A, B, the *ray* $\overrightarrow{AB}$ consists of the points A, B and of all the points C on the line $\overleftrightarrow{AB}$ such that $A - C - B$ or $A - B - C$.
5. A set of points R is called *convex* if for any two distinct points A, B in R, the segment $\overline{AB}$ is contained in R.
6. We say that lines ℓ and m are *parallel* if there does not exist a point that is on both ℓ and m.
7. Let A, B, C, and D be points. We say that segments $\overline{AB}$ and $\overline{CD}$ are *congruent* if $d(A, B) = d(C, D)$, and we denote this by writing $\overline{AB} \cong \overline{CD}$.
8. If l is a line in a plane P, then by SMSG Postulate 9, the complement of l in P is a union of two convex disjoint sets H_1 and H_2 such that for any points $A \in H_1$ and $B \in H_2$, the segment $\overline{AB}$ intersects l at a single point. The sets H_1 and H_2 are called *half-planes* of P determined by l.
9. Let A, B, and C be non-collinear points. Then the angle $\angle ABC$ is the union of $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called the *sides* of $\angle BAC$ and the point A is called the *vertex* of $\angle BAC$.
10. Let A, B, C, D, E, and F be points. We say that angles $\angle ABC$ and $\angle DEF$ are *congruent* if $m\angle ABC = m\angle DEF$, and we denote this by writing $\angle ABC \cong \angle DEF$.
11. Let A, B, C, D, E, and F be points. We say that triangles $\triangle ABC$ and $\triangle DEF$ are *congruent*, which we denote by $\triangle ABC \cong \triangle DEF$, if the following hold:
$$\angle A \cong \angle D, \quad \angle B \cong \angle E, \quad \angle C \cong \angle F, \quad \overline{AB} \cong \overline{DE}, \quad \overline{BC} \cong \overline{EF}, \quad \text{and} \quad \overline{CA} \cong \overline{FD}.$$
12. If A, B, C are three distinct non-collinear points, then the *interior of angle* $\angle BAC$ consists of the points on $\overrightarrow{AB} \cup \overrightarrow{AC}$, and of all points D that lie between a point on $\overrightarrow{AB}$ and a point on $\overrightarrow{AC}$.
13. An angle is *obtuse* if it has measure more than $90°$. An angle is acute if it has measure less than $90°$. An angle is right if it has measure $90°$.
14. Two angles are *adjacent* if they have a common side and the intersection of their interiors is that common side. (Recall that we, perhaps regrettably, defined the interior to include the rays making up the angle.)

15. If two angles are adjacent, and if the union of the non-shared sides is a line, then we say that the angles form a *linear pair*.
16. Two angles are *vertical* if they have a common vertex, their intersection is equal to that vertex and if the sides of each angle can be ordered in such a way that the union of the first sides of these angles is a line and the union of the second sides of these angles is a line.
17. Two angles are *supplementary* if the sum of their measures is $180°$. Two angles are *complementary* if the sum of their measures is $90°$.
18. The *midpoint* of a line segment is a point on the segment that is equidistant from the endpoints of the segment.
19. A *bisector* of an angle $\angle BAC$ is a ray $\overrightarrow{AD}$ that is contained in $\angle BAC$ and such that $m\angle BAD = m\angle DAC = \frac{1}{2}m\angle BAC$.
20. Two lines l_1, l_2 are said to be *perpendicular* if they intersect at a point A such that for any point B on l_1 and any point C on l_2 such that $B \neq A$ and $C \neq A$, the angle $\angle BAC$ is $90°$.
21. If A, B, C are three non-collinear points, then a *triangle* $\triangle ABC$ is the union of the segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
22. If A, B, C are three non-collinear points, then we define the *interior* of $\triangle ABC$ to be the intersection of the interiors of the angles $\angle BAC$, $\angle ACB$, and $\angle CBA$.
23. Let A, B, C, and D be distinct points, and assume that no two of the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ intersect at any point other than A, B, C, or D. Then the *quadrilateral* $ABCD$ is defined to be $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DA}$.