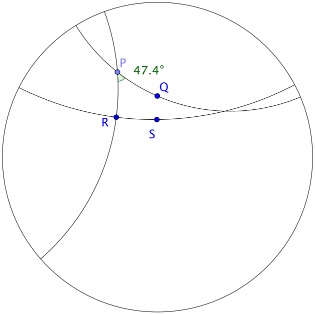
Some notes from class

2018-03-26

Set-up for the next few results

The "usual set-up"

Let R and S be points with $R \neq S$, and suppose that P is a point not on $\overleftrightarrow{RS}$ with $\overleftrightarrow{PR} \perp \overleftrightarrow{RS}$. We will typically be interested in rays of the form $\overrightarrow{PQ}$, where Q is a point on the same side of $\overleftrightarrow{PR}$ as S.



d_0

Let d_0 stand for the least upper bound of the set of all angle measures $m(\angle RPQ)$ for which $\overrightarrow{PQ}$ intersects $\overrightarrow{RS}$.

What is d_0 really?

Theorem

Assume the usual about S, R, P, Q, and d_0. Then
(i) $d_0 \notin D$, and
(ii) if $m(\angle RPQ) < d_0$, then $\overrightarrow{PQ}$ intersects $\overrightarrow{RS}$.

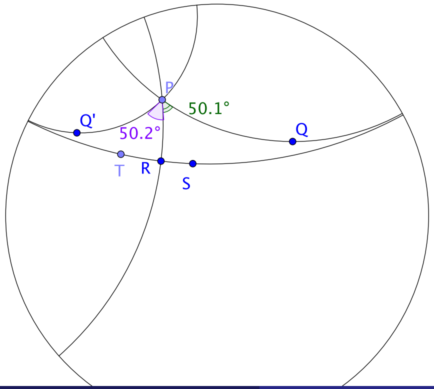
Definition

Assume S, R, P, Q and d_0 as usual. If $m(\angle RPQ) = d_0$, we say that $\angle RPQ$ is the angle of parallelism for P and $\overrightarrow{RS}$.

Symmetry

Theorem

Let T, R, and S be collinear with $T - R - S$. Assume that
(i) $\angle RPQ$ is the angle of parallelism for P and $\overrightarrow{RS}$,
(ii) $\angle RPQ'$ is the angle of parallelism for P and $\overrightarrow{RT}$.
Then $\angle RPQ \cong \angle RPQ'$.



Symmetry

Proof (prev result). If $m(\angle RPQ') \neq m(\angle RPQ)$, assume, without loss of generality, that $m(\angle RPQ') < m(\angle RPQ)$. Then there exists a point Q'' in the interior of angle $\angle RPQ$ such that $\angle RPQ' \cong \angle RPQ''$. Since $m(\angle RPQ'') < m(\angle RPQ)$, we know (from an earlier theorem) that $\overrightarrow{PQ''}$ must intersect $\overrightarrow{RS}$ at some point V. Let $W \in \overrightarrow{RT}$ with $WR = RV$, and note that SAS implies that $\triangle VRP \cong \triangle WRP$. This implies that $\angle RPW \cong \angle RPV$. On the other hand, we have $\angle RPV = \angle RPQ''$, so $\angle RPW \cong \angle RPQ'' \cong \angle RPQ'$. This forces $\overrightarrow{PQ'} = \overrightarrow{PW}$. But this implies that $\overrightarrow{PQ'}$ intersects $\overrightarrow{RT}$, which contradicts the fact that we have assumed $\angle RPQ'$ to be the angle of parallelism for P and $\overrightarrow{RT}$. The contradiction resulted from the assumption that $m(\angle RPQ') < m(\angle RPQ)$, so that cannot be the case.

If angle of parallelism < 90

Theorem

If the angle of parallelism for a given line ℓ and point $P \notin \ell$ is less than 90, then there exist at least two lines on P parallel to ℓ.

If angle of parallelism < 90

Theorem

Suppose, for a given line ℓ and point $P \notin \ell$, that there exist at least two lines on P parallel to ℓ. Then the angle of parallelism for P and ℓ is less than 90.