Some notes from class

2018-03-23

Least upper bound

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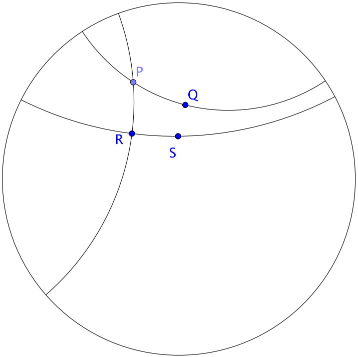
Let S be a subset of $\mathbb{R}$. A number $b \in \mathbb{R}$ is called a least upper bound for S if

1. $s \leq b$ for all $s \in S$ (i.e. b is an upper bound for S),
2. if $c < b$, then c is not an upper bound for S.

Set-up for the next few results

The "usual set-up"

Let R and S be points with $R \neq S$, and suppose that P is a point not on $\overleftrightarrow{RS}$ with $\overleftrightarrow{PR} \perp \overleftrightarrow{RS}$. We will typically be interested in rays of the form $\overrightarrow{PQ}$, where Q is a point on the same side of $\overleftrightarrow{PR}$ as S.

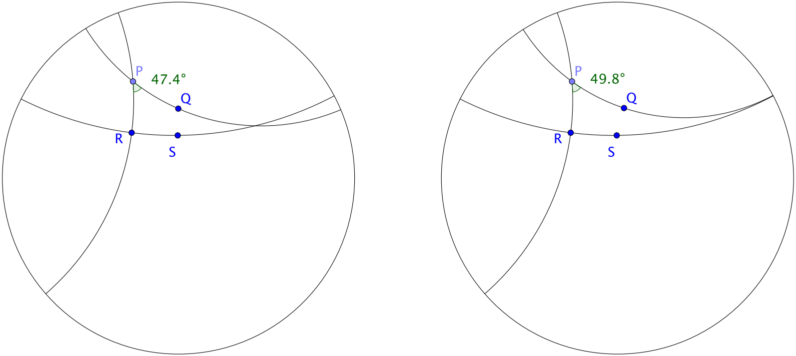


Angle of parallelism

Let

$$D = \{m(\angle RPQ) \mid \overrightarrow{PQ} \text{ intersects } \overrightarrow{RS}\}$$

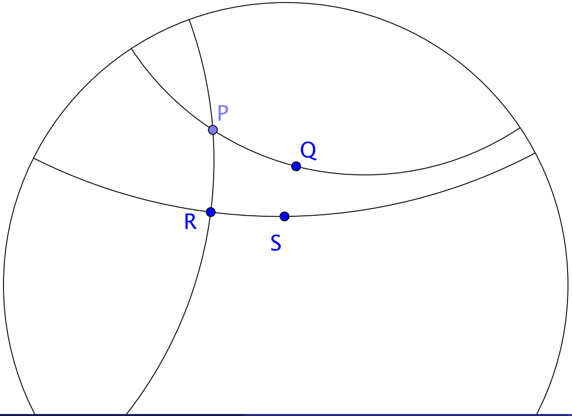
(Note D is bounded above, so it has a LUB.) Let d_0 be the least upper bound for D in $\mathbb{R}$.



What is d_0 really?

Theorem

Assume the usual about S, R, P, Q, and d_0. Then
(i) $d_0 \notin D$, and
(ii) if $m(\angle RPQ) < d_0$, then $\overrightarrow{PQ}$ intersects $\overrightarrow{RS}$.



The angle of parallism

Def

Assume S, R, P, Q and d_0 as usual. If $m(\angle RPQ) = d_0$, we say that $\angle RPQ$ is the angle of parallelism for P and $\overrightarrow{RS}$.

Theorem

Let $T, R,$ and S be collinear wit $T - R - S$. Assume that
(i) $\angle RPQ$ is the angle of parallelism for P and $\overrightarrow{RS}$,
(ii) $\angle RPQ'$ is the angle of parallelism for P and $\overrightarrow{RT}$.
Then $\angle RPQ \cong \angle RPQ'$.