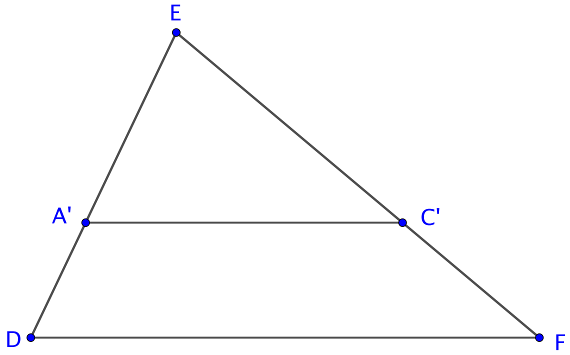
Some notes from class

2018-03-16

In the middle of proving AAA

Assuming $\triangle ABC$ and $\triangle DEF$ with $\angle A \cong \angle D$, etc. . .



Cool theorems

Theorem (A)

The three medians of a triangle are concurrent.

Theorem (B)

The three angle bisectors of a triangle are concurrent.

Theorem (C)

The three perpendicular bisectors of a triangle are concurrent.

Theorem (D)

The three altitudes of a triangle are concurrent.

Cool theorems

Theorem (A)

The three medians of a triangle are concurrent.

DONE! (Proved this earlier.)

Theorem (B)

The three angle bisectors of a triangle are concurrent.

Proof. Basically, angle bisectors are equidistant from sides. Work two sides at a time.
Label the vertices of the triangle A, B, and C. Let ray q be the angle bisector of $\angle A$, and let r be the angle bisector of $\angle B$. By the Crossbar Theorem, q intersects side $\overline{BC}$, and also r intersects ray q at some point D. Let $P \in \overline{AB}$, $Q \in \overline{BC}$, and $R \in \overline{CA}$ such that $\overline{DP} \perp \overline{AB}$, $\overline{DQ} \perp \overline{BC}$, and $\overline{DR} \perp \overline{CA}$.

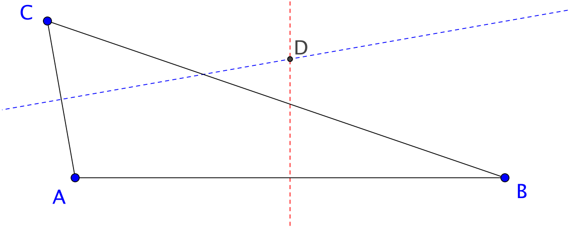
Since D is on q, it follows (from a lemma) that $DP = DR$. Since D is on r, it follows that $DP = DQ$. Therefore $DQ = DR$, and D is consequently equidistant from $\overline{BC}$ and $\overline{CA}$. This forces D (by our lemma) to be on the angle bisector of $\angle C$.

Cool theorems

Theorem (C)

The three perpendicular bisectors of a triangle are concurrent.

Proof. Basically, perpendicular bisectors of segments are equidistant from the endpoints. Plan: Consider one segment (two endpoints) at a time.



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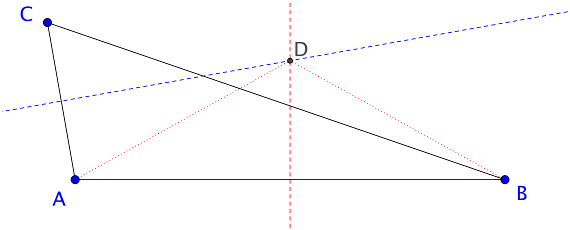

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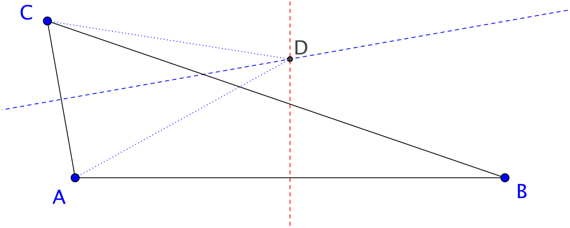

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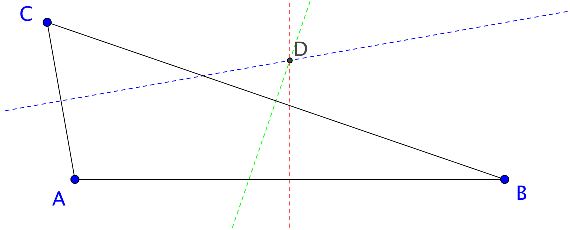
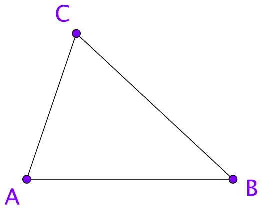


Cool theorems

Theorem (D)

The three altitudes of a triangle are concurrent.

Proof. Construct a bunch of parallels. Cite previous theorem.



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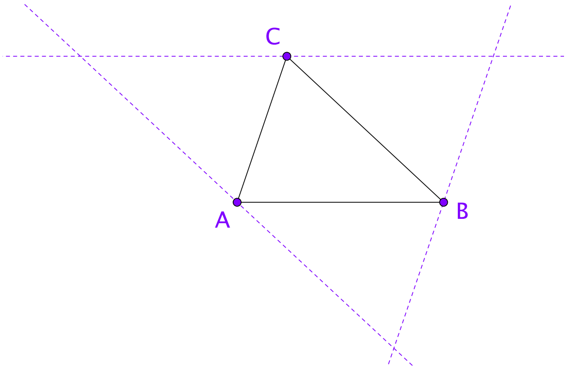

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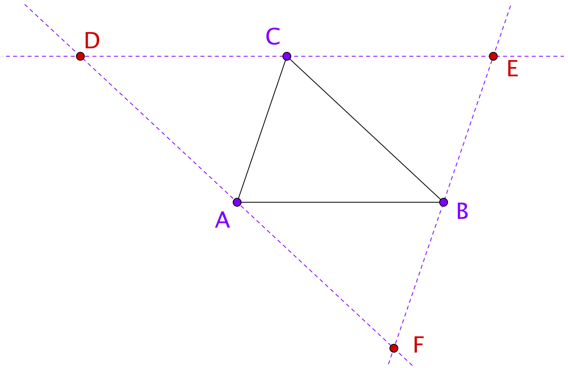

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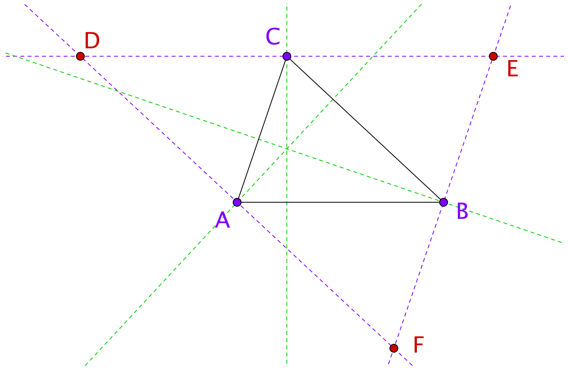

A proof of the "altitudes" theorem.

Theorem (D)

The three altitudes of a triangle are concurrent.

Proof. Let ℓ be the line on A parallel to $\overleftrightarrow{CB}$, let m be the line on B parallel to $\overleftrightarrow{AC}$, and let n be the line on C parallel to $\overleftrightarrow{AB}$. Let D be the intersection of ℓ and n, let E be the intersection of m and n, and let F be the intersection of ℓ and m.
Note that, by construction, $\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{BC}$ and $\overleftrightarrow{DC}$ is parallel to $\overleftrightarrow{AB}$. Thus $\square ABCD$ is a parallelogram and $AB = CD$ (from basic facts about parallelograms). Similar reasoning shows $\square ABEC$ is a parallelogram and $CE = AB$. Thus C is the midpoint of side $\overline{DE}$ of $\triangle DEF$. Similar reasoning shows B is the midpoint of side $\overline{EF}$ and A is the midpoint of side $\overline{DF}$.

$\vdots$