Some notes from class

2018-03-12

Area Axioms

17. (Area) To every polygonal region, there corresponds a unique positive number called the *area* of the region.
18. (Congruence versus Area) If two triangles are congruent, then the triangular regions have the same area.
19. (Additivity of Area) Suppose that the region R is the union of two regions R_1 and R_2. Suppose also that R_1 and R_2 intersect at most in a finite number of segments and points. Then the area of R is the sum of the areas of R_1 and R_2.
20. (Area of Rectangle) The area of a rectangle is equal to the product of the length of its base and the length of its altitude.

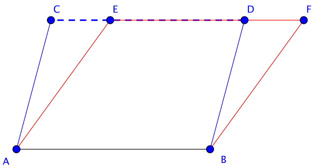
Some theorems we have proved

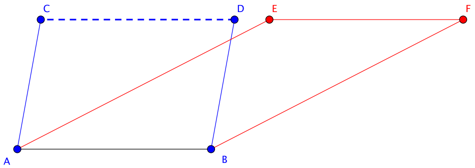
Theorem (Euclidean geometry)

Let $\overleftrightarrow{AM}$ be a median of $\triangle ABC$ with $M \in \overline{BC}$. If $\overleftrightarrow{CN}$ is a median, with $n \in \overline{AB}$, then $\overleftrightarrow{CN}$ intersects $\overline{AM}$ at a point $\frac{2}{3}$ of the way from A to M.

Theorem (Euclidean geometry)

Suppose $\Diamond ABDC$ is a parallelogram, and let E and F be points on line $\overleftrightarrow{CD}$ with $EF = CD$. Then the area of $\Diamond ABDC$ is equal to the area of $\Diamond ABFE$.





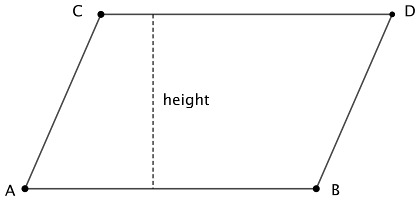
Summary - parallelograms

Theorem (Euclidean geometry)

Suppose $\Diamond ABDC$ *is a parallelogram, and let* E *and* F *be points on line* $\overleftrightarrow{CD}$ *with* $EF = CD$. *Then the area of* $\Diamond ABDC$ *is equal to the area of* $\Diamond ABFE$.

Corollary

The area of a parallelogram is the product of the length of its base and its height.



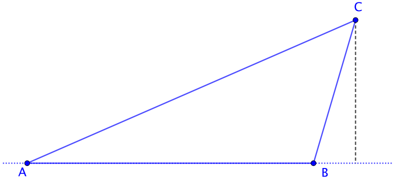
Triangles

Note

A right triangle is half of a rectangle, so its area is obvious.

Theorem (Euclidean geometry)

Suppose $\triangle ABC$ is any triangle. Then the area of $\triangle ABC$ equals one half the product of AB and the distance from C to $\overleftrightarrow{AB}$. (Keep in mind that the distance from C to $\overleftrightarrow{AB}$ is defined to be...)

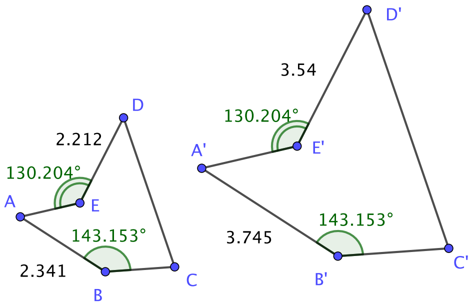


Similarity (Sec 4.4)

Definition

Two polygons $P_1P_2\dots P_n$ and $Q_1Q_2\dots Q_n$ are said to be *similar* under the correspondence $P_i \leftrightarrow Q_i$ if:

1. $\angle P_i \cong \angle Q_i$ for all $i \in \{1, 2, \dots, n\}$
2. $\frac{P_1P_2}{Q_1Q_2} = \frac{P_2P_3}{Q_2Q_3} = \cdots = \frac{P_{n-1}P_n}{Q_{n-1}Q_n}$.



First step toward triangle similarity condition(s)

Theorem

Given $\triangle ABC$*, let* $D \in \overline{AC}$*, and let* $E \in \overline{BC}$ *with* $\overleftrightarrow{DE}$ *parallel to* $\overleftrightarrow{AB}$*. Then* $\frac{CD}{DA} = \frac{CE}{EA}$.