Some notes from class

2018-02-23

Working toward the Saccheri-Legendre Theorem

Definition

A quadrilateral is a *rectangle* if every interior angle measure is 90°.

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A quadrilateral is a *Saccheri quadrilateral* if there exist two congruent opposite sides (called legs), and one of the remaining sides (called the base) is perpendicular to both legs.

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A quadrilateral is a *Lambert quadrilateral* if it contains three 90° angles.

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Theorem

If $\lozenge ABCD$ is a Saccheri quadrilateral with base $\overline{AB}$, then $\triangle ABC \cong \triangle BAD$ and $\triangle DCB \cong \triangle CDA$. In particular, the diagonals are congruent.

Results from last time

Theorem

In a Saccheri quadrilateral, the length of the summit is greater than or equal to the length of the base.

Theorem

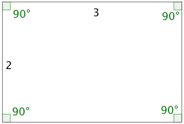
In a Saccheri quadrilateral, the line joining the midpoint of the base to the midpoint of the summit is perpendicular to both. Also, the summit is parallel to the base.

Theorem

In a Lambert quadrilateral, the length of a side between two right angles is less than or equal to the length of the opposite side.

Question: Assume geometry is currently *neutral*

Suppose this rectangle exists. Can you make one that is 5×7?



Summary of previous slide

To save time, we assume the following (without writing proofs).

Theorem

If a rectangle exists, then there is a rectangle whose sides are arbitrarily large. (i.e. They are at least as long as some given lengths.)

Theorem

If a rectangle exists, then for any given length and width, there is a rectangle having exactly those dimensions.

New result(s)

Theorem

If a rectangle exists, then every triangle has an angle sum of $180°$.
We prove a slightly easier result: If a rectangle exists, then every right triangle has an angle sum of 180.

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Theorem (Converse of previous theorem)

If every (right) triangle has an angle sum of $180°$*, then a rectangle exists.*

Key connection between triangles and parallel lines

Theorem

The Euclidean parallel postulate (Playfair's postulate) is equivalent to the statement that the angle sum of every triangle is $180°$*.*

$\Longrightarrow$ Assume Playfair's postulate holds, and prove that every triangle has an angle sum of $180°$.

$\Longleftarrow$ Assume every triangle has an angle sum of $180°$, and prove that given any line ℓ and a point P not on ℓ, there is …

The tricky direction

$\Longleftarrow$ Assume every triangle has an angle sum of $180°$, and prove that given any line ℓ and a point P not on ℓ, there is at most one parallel line on P.