Some notes from class

2018-02-21

Working toward the Saccheri-Legendre Theorem

Definition

A quadrilateral is a *rectangle* if every interior angle measure is 90°.

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A quadrilateral is a *Saccheri quadrilateral* if there exist two congruent opposite sides (called legs), and one of the remaining sides (called the base) is perpendicular to both legs.

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A quadrilateral is a *Lambert quadrilateral* if it contains three 90° angles.

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Theorem

If $\Diamond ABCD$ *is a Saccheri quadrilateral with base* $\overline{AB}$*, then* $\triangle ABC \cong \triangle BAD$ *and* $\triangle DCB \cong \triangle CDA$*. In particular, the diagonals are congruent.*

New result

Theorem

In a Saccheri quadrilateral, the length of the summit is greater than or equal to the length of the base.

Proof. Hint: Use the Hinge Theorem.

New result

Theorem

In a Saccheri quadrilateral, the line joining the midpoint of the base to the midpoint of the summit is perpendicular to both. Also, the summit is parallel to the base.

Proof. Hint: Connect midpoint of base to each of the summit vertices.

New result

Theorem

In a Lambert quadrilateral, the "unknown angle" has a measure of at most $90°$.

Proof. Hint: Follows from Saccheri-Legendre Theorem.

New result

Theorem

In a Lambert quadrilateral, the length of a side between two right angles is less than or equal to the length of the opposite side.

New result

Theorem

If a rectangle exists, then there is a rectangle whose sides are arbitrarily large. (i.e. They are at least as long as some given lengths.)

New result

Theorem

If a rectangle exists, then for any given length and width, there is a rectangle having exactly those dimensions.

New result

Theorem

If a rectangle exists, then every triangle has an angle sum of $180°$*. (To prove, first show for right triangles, and then extend result to arbitrary triangles.)*