# Some notes from class

2018-02-16

# Ways to recognize Euclidean geometry

1. Playfair's Axiom holds. (This is our official assumption.)
2. Euclid's 5th postulate holds.
3. The converse of the AIAT holds.
4. If a line intersects one of two parallel lines, then it intersects the other.
5. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

# Working toward the Saccheri-Legendre Theorem

## Lemma

The sum of the measures of any two interior angles of a triangle is less than 180.

## Lemma (Triangle Smushing)

Given a triangle $\triangle ABC$, there exists a triangle $\triangle A'B'C'$ with $m(\angle A) + m(\angle B) + m(\angle C) = m(\angle A') + m(\angle B') + m(\angle C')$ and $m(\angle A') \leq \frac{1}{2}m(\angle A)$.

# Working toward the Saccheri-Legendre Theorem

## Theorem

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*Sketch of proof.*

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Suppose, for the sake of contradiction, that there is some $\triangle ABC$ with interior angle sum of $180.3°$. Construct:

1. $\triangle A_1B_1C_1$ same angle sum, $m(\angle A_1) \leq \frac{1}{2}m(\angle A)$

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2. $\triangle A_2B_2C_2$ same angle sum, $m(\angle A_2) \leq \frac{1}{2}m(\angle A_1)$

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2. $\triangle A_2B_2C_2$ same angle sum, $m(\angle A_2) \leq \frac{1}{2}m(\angle A_1)$
3. $\triangle A_3B_3C_3$ same angle sum, $m(\angle A_3) \leq \frac{1}{2}m(\angle A_2)$

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2. $\triangle A_2B_2C_2$ same angle sum, $m(\angle A_2) \leq \frac{1}{2}m(\angle A_1)$
3. $\triangle A_3B_3C_3$ same angle sum, $m(\angle A_3) \leq \frac{1}{2}m(\angle A_2)$
4. $\triangle A_4B_4C_4$ same angle sum, $m(\angle A_4) \leq \frac{1}{2}m(\angle A_3)$
5. $\vdots$

**Think about** $m(\angle B_7) + m(\angle C_7)$.