Some notes from class

2018-02-14

Euclid's 5th Postulate

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Suppose that two lines are intersected by a transversal in such a way that the sum of the measures of two interior angles on the same side of the transversal is less than 180°. Then the two lines intersect on that side of the transversal.

Playfair's Postulate

If ℓ is a line and P is a point not on ℓ, then there is at most one line on P that is parallel to ℓ.

Alternate Interior Angle Theorem

Theorem (Alternate Interior Angle Theorem)

If two lines are intersected by a transversal forming a pair of congruent alternate interior angles, then the lines are parallel.

Theorem (Alternate Interior Angle Theorem)

Suppose that two lines are intersected by a transversal. If a pair of alternate interior angles are congruent, then the lines are parallel.

Theorem (Converse of Alternate Interior Angle Theorem)

Suppose that two lines are intersected by a transversal. If the lines are parallel, then a pair of alternate interior angles are congruent.

Playfair $\iff$ Converse of AIAT

Theorem (Playfair $\implies$ Converse of AIAT)

Suppose Playfair's Postulate holds, and assume that two lines are intersected by a transversal. If the lines are parallel, then a pair (in fact, all pairs) of alternate interior angles are congruent.

Proof. Let ℓ and m be parallel lines, and let t be a transversal. We must show that t forms a pair of congruent alternate interior angles. Assume t intersects ℓ at point P and m at point Q. Let R be a point on m with $R \neq Q$, and let $S \neq P$ be a point on ℓ in the opposite half-plane (as determined by t) from R. We will show that $\angle RQP \cong \angle SPQ$.

By the angle construction postulate, there is a point T in the half-plane containing S (formed from t) such that $m(\angle QPT) = m(\angle RQP)$. Then the AIAT implies that $\overleftrightarrow{PT}$ is parallel to $\overleftrightarrow{QR}$. But since ℓ (same as $\overleftrightarrow{PS}$) is a line on P parallel to m, Playfair's Postulate implies that $\overleftrightarrow{PT} = \ell$.

Ways to recognize Euclidean geometry

1. Playfair's Axiom holds. (This is our official assumption.)
2. Euclid's 5th postulate holds.
3. The converse of the AIAT holds.

Ways to recognize Euclidean geometry

1. Playfair's Axiom holds. (This is our official assumption.)
2. Euclid's 5th postulate holds.
3. The converse of the AIAT holds.
4. If a line intersects one of two parallel lines, then it intersects the other.
5. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.