Some notes from class

2018-01-31

Theorem from Sec 3.2

Theorem

(i) Every segment has a unique midpoint.

- Names of points: A and B
- Line $\overleftrightarrow{AB}$
- Points on $\overleftrightarrow{AB}$ correspond to real numbers
- $A \longleftrightarrow x_A \in \mathbb{R} \qquad B \longleftrightarrow x_B \in \mathbb{R}$
- Let $e = \frac{x_A + x_B}{2} \in \mathbb{R}$, and let $E \in \overleftrightarrow{AB}$ with $E \longleftrightarrow e$.
- $d(A, E) = |x_A - \frac{x_A+x_B}{2}| = \frac{|x_A - x_B|}{2}$ and $d(E, B) = \cdots = \frac{|x_B - x_A|}{2}$.

- Suppose that $C \in \overleftrightarrow{AB}$ is also a midpoint. Let $C \longleftrightarrow c \in \mathbb{R}$.
- $d(A, C) = d(C, B) \implies |x_A - c| = |c - x_B| \implies x_A - c = \pm(c - x_B)$

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Suppose that C is any midpoint of $\overline{AB}$ (potentially not equal to E). By Axiom ____, let c be the real number corresponding to the point C on $\overleftrightarrow{AB}$. Then $d(A, C) = d(C, B)$, so $|x_A - c| = |c - x_B|$. This forces $x_A - c = \pm(c - x_B)$. If $x_A - c = -(c - x_B)$, then $x_A = x_B$, which forces the contradiction $A = B$. If $x_A - c = c - x_B$, then $c = \frac{x_A + x_B}{2}$, so...

A basic theorem

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If two distinct lines intersect, their intersection is exactly one point.

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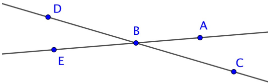
Proof. Let ℓ_1 and ℓ_2 be distinct lines, and let P and Q be points of intersection of ℓ_1 and ℓ_2. If $P \neq Q$, then $\ell_1 = \ell_2$ by Postulate 1 (that two points determine a line). Thus it must be that $P = Q$, and it follows there is at most one point of intersection. $\square$

Theorem

Vertical angles are congruent.

Proof. Suppose we are given a pair of vertical angles. Then by definition, we may write the angles as $\angle ABC$ and $\angle DBE$, where we assume



the points A, B, C, D, E satisfy $A - B - E$ and $C - B - D$. Since C, B, and D are collinear, the angles $\angle CBA$ and $\angle ABD$

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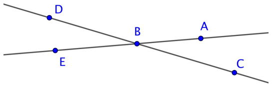

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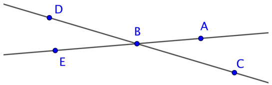

the points A, B, C, D, E satisfy $A - B - E$ and $C - B - D$. Since C, B, and D are collinear, the angles $\angle CBA$ and $\angle ABD$ form a linear pair. By the supplement postulate, they are supplementary, so $m\angle CBA + m\angle ABD = 180$.

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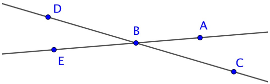

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$m\angle CBA + m\angle ABD = 180$. Similar reasoning applied to $\angle EBD$ and $\angle DBA$ implies that $m\angle EBD + m\angle DBA = 180$.

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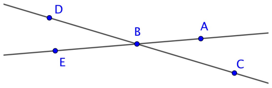

the points A, B, C, D, E satisfy $A - B - E$ and $C - B - D$. Since C, B, and D are collinear, the angles $\angle CBA$ and $\angle ABD$ form a linear pair. By the supplement postulate, they are supplementary, so $m\angle CBA + m\angle ABD = 180$. Similar reasoning applied to $\angle EBD$ and $\angle DBA$ implies that $m\angle EBD + m\angle DBA = 180$. Thus $m\angle CBA = 180 - m\angle ABD = m\angle EBD$, as desired. □

1882 was a good year

Theorem

Suppose a line ℓ intersects $\triangle PQR$ at a point S such that $P - S - Q$. Then ℓ intersects $\overline{PR}$ or $\overline{RQ}$.

Proof.

Soon we will prove...

...that the base angles of an isosceles triangle are congruent.

Start of argument. Let $\triangle ABC$ be a triangle with side $\overline{AB}$ congruent to side $\overline{AC}$. Draw the angle bisector of $\angle A$ and let D be the point at which it meets side $\overline{BC}$ ...

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Theorem (Crossbar Theorem)

If X is a point in the interior of $\triangle ABC$, then $\overrightarrow{AX}$ intersects $\overline{BC}$ in a point Y such that $B - Y - C$.

Definition

If $\triangle ABC$ is a triangle, we define the *interior* of $\triangle ABC$ to be the collection of all points not on $\overline{AB}$ or $\overline{BC}$ or $\overline{CA}$ that belong to the intersection of the interiors of angles $\angle A$, $\angle B$, and $\angle C$.

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Proof.