Some notes from class

2018-01-29

Our First Theorem

Theorem

(i) Every segment has a unique midpoint.
(ii) Every angle has a unique bisector.

Proof outline for part (i).

- Notation
- Show the existence of a midpoint.
- Show that the midpoint is unique. (There is only one.)

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- $d(A, E) = |x_A - \frac{x_A + x_B}{2}| = \frac{|x_A - x_B|}{2}$ and $d(E, B) = \cdots = \frac{|x_B - x_A|}{2}$.

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- Suppose that $C \in \overleftrightarrow{AB}$ is also a midpoint. Let $C \longleftrightarrow c \in \mathbb{R}$.
- $d(A, C) = d(C, B) \implies |x_A - c| = |c - x_B| \implies x_A - c = \pm(c - x_B)$

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Let A and B be distinct points. By Axiom ???, the points on $\overleftrightarrow{AB}$ are in one-to-one correspondence with the real numbers, so we may write x_A and x_B for real numbers corresponding to A and B respectively.

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Suppose that C is any midpoint of $\overline{AB}$ (potentially not equal to E). By Axiom ???, let c be the real number corresponding to the point C on $\overleftrightarrow{AB}$. Then $d(A, C) = d(C, B)$, so $|x_A - c| = |c - x_B|$. This forces $x_A - c = \pm(c - x_B)$. If $x_A - c = -(c - x_B)$, then $x_A = x_B$, which forces the contradiction $A = B$. If $x_A - c = c - x_B$, then $c = \frac{x_A + x_B}{2}$, so…