Some notes from class

2018-01-22

SMGS Axioms: (point, line, on)

1. (Two Points Determine a Line) Given any two different points, there is exactly one line that contains them both.
2. (Distance Postulate) To every pair of different points, there corresponds a unique positive number.
   For points, A and B, this unique positive number is denoted by $d(A, B)$, and is referred to as the distance between A and B.
3. (Ruler Postulate) The points of a line can be put into one-to-one correspondence with the real numbers in such a way that
   i. to every point, there corresponds exactly one real number called the point's *coordinate*
   ii. to every real number, there corresponds exactly one point of the line, and
   iii. the distance between two points is the absolute value of the difference of the corresponding coordinates.

SMSG Axioms

4. (Ruler Placement Postulate) Given two points P and Q of a line, the coordinate system (i.e. the one-to-one correspondence) can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

5-8. Postulates 5–8 deal with geometry of three dimensions, and we ignore them here.

9. (Plane Separation) Given a line ℓ and a plane containing it, the points of the plane α that do not lie on the line form two nonempty sets such that
   i. each of the sets is convex, and
   i. if point P is in one set and point Q is in the other, then $\overline{PQ} \cap \ell \neq \emptyset$.

10. (Space Separation) We ignore this postulate for now (3-D).

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11. (Angle Measurement) To every angle $\angle ABC$, there corresponds a unique real number between 0 and 180, which we denote by $m\angle ABC$.
12. (Angle Construction) Let $\overrightarrow{AB}$ be a ray on the edge of half-plane H. For every number r between 0 and 180, there is exactly one ray $\overrightarrow{AP}$, with P in H such that $m\angle PAB = r$.
13. (Angle Addition) If D is a point in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.
14. (Supplement) If two angles form a linear pair, then they are supplementary.

SMSG Axioms

15. (SAS Congruence for Triangles) Suppose we are given a correspondence of vertices and sides between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence of triangles.
16. (Parallel Postulate) Given a line ℓ and a point P not on ℓ, there is at most one line through P that is parallel to ℓ.
    (Assuming this postulate makes our geometry *Euclidean*.)

SMSG Axioms

17. (Area) To every polygonal region, there corresponds a unique positive number called the *area* of the region.
18. (Congruence versus Area) If two triangles are congruent, then the triangular regions have the same area.
19. (Additivity of Area) Suppose that the region R is the union of two regions R_1 and R_2. Suppose also that R_1 and R_2 intersect at most in a finite number of segments and points. Then the area of R is the sum of the areas of R_1 and R_2.