Integrals related to paths

Formula	**Name**	**Comment(s)**
$\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$	Curve	
$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$	Length of curve	Often write $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$
$\int_C f(x, y, z)\, ds$ (Recall formula for ds.)	Line integral of scalar function over C	*Weighted* length. Can view as area of fence
$\int_C \mathbf{F} \bullet \mathbf{T}\, ds \quad (= \int_C \mathbf{F} \bullet d\mathbf{r})$	Line integral of vector field over C	Gives Work = Force × Distance

Integrals related to surfaces

$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle$	Surface	
$\iint_D \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\, du\, dv$	Surface area	Can replace $du\, dv$ with $dv\, du$ or dA. We often write $dS = \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\, dA$.
$\iint_S f(x, y, z)\, dS$ $\iint_D f(\mathbf{r}(u, v))\, \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\, dA$	Surface integral of f over S	*Weighted* surface area. Could give mass if $f(x, y, z)$ gives density.
$\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$	Surface integral of vector field $\mathbf{F}$ over surface S	$\mathbf{n}$ = unit normal vector = $\frac{\mathbf{r}_u \times \mathbf{r}_v}{\lvert\mathbf{r}_u \times \mathbf{r}_v\rvert}$. We call this the *flux* of $\mathbf{F}$ across S. (Useful in fluid flow and in electricity and magnetism.)