List of Axioms

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Incidence axioms

- (Axiom I - 1) Each two distinct points determine a line.
  Note: This allows us to write $\overleftrightarrow{AB}$.
- (Axiom I - 2) Three noncolinear points determine a plane.
- (Axiom I - 3) If two points lie in a plane, then any line containing those two points lies in that plane.
- (Axiom I - 4) If two distinct planes meet, their intersection is a line.
- (Axiom I - 5) Space consists of at least four noncoplanar points, and contains three noncolinear points. Each plane is a set of points of which at least three are noncolinear, and each line is a set of at least two distinct points.

Distance axioms

- (Axiom D-1) Each pair of points A and B is associated with a unique real number, called the **distance** from A to B, denoted by AB.
- (Axiom D-2) For all points A and B, $AB \geq 0$, with equality only when $A = B$.
- (Axiom D-3) For all points A and B, $AB = BA$.
- (Axiom D-4, Ruler Postulate) The points of each line ℓ may be assigned to the entire set of real numbers (we call the corresponding real numbers *coordinates*) in such a manner that
  - each point on ℓ is assigned to a unique coordinate,
  - no two points are assigned to the same coordinate,
  - any two points on ℓ may be assigned the coordinates 0 and some positive real number, respectively,
  - if points A and B on ℓ have coordinates a and b, then $AB = |a - b|$.

Angle axioms

- (Axiom A-1, existence of angle measure) Each angle $\angle ABC$ is associated with a unique real number between 0 and 180, called its *measure* and denoted $m\angle ABC$. No angle can have measure 0 or 180.
- (Axiom A-2, angle addition postulate) If D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$. Conversely, if $m\angle ABD + m\angle DBC = m\angle ABC$, then ray $\overrightarrow{BD}$ passes through an interior point of angle $\angle ABC$.

Angle axioms, continued

- (Axiom A-3, protractor postulate) The set of rays $\overrightarrow{AX}$ lying on one side of a given line $\overleftrightarrow{AB}$, including ray $\overrightarrow{AB}$, may be assigned to the entire set of real numbers x $(0 \leq x < 180)$, called coordinates, in such a manner that
  1. each ray is assigned to a unique coordinate
  2. no two rays are assigned to the same coordinate
  3. the coordinate of $\overrightarrow{AB}$ is 0
  4. if rays $\overrightarrow{AC}$ and $\overrightarrow{AD}$ have coordinates c and d, then $m\angle CAD = |c - d|$.
- (Axiom A-4, linear pair axiom) A linear pair of angles is a supplementary pair.

Plane separation

- (Axiom H-1, plane separation) Let ℓ be a line lying in a plane $\mathbf{P}$. The set of all points in $\mathbf{P}$ not on ℓ consists of the union of two subsets H_1 and H_2 of $\mathbf{P}$ such that:
  1. H_1 and H_2 are convex sets,
  2. H_1 and H_2 have no points in common,
  3. If $A \in H_1$ and $B \in H_2$, then the line ℓ intersects $\overline{AB}$.

Congruence

- (Axiom C-1, SAS) If two sides and the included angle of one triangle are congruent, respectively, to two sides and the included angle of another triangle, then the two triangles are congruent under the given correspondence between sides and angles.

Parallelism

- (Axiom P-1, Playfair's Axiom) Let ℓ be a line, and P a point not on ℓ. Then there is exactly one line in the plane containing P and ℓ that contains P and is parallel to ℓ.