Some notes from class

2018-04-11

The Taylor series for $f(x)$ is...

$$T(x) = \frac{f(a)}{0!} + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3$$

$$+ \frac{f^{(4)}(a)}{4!}(x-a)^4 + \frac{f^{(5)}(a)}{5!}(x-a)^5 + \frac{f^{(6)}(a)}{6!}(x-a)^6 \cdots$$

Example 1:

Construct the Taylor series for $f(x) = \ln(1+x)$ centered at $a = 0$.

$f(x) = \ln(1+x)$

$f'(x) = (1+x)^{-1}$

$f''(x) = (-1)(1+x)^{-2}$

$f^{(3)}(x) = (2)(1)(1+x)^{-3}$

$f^{(4)}(x) = (-1)(3)(2)(1)(1+x)^{-4}$

$f^{(5)}(x) = (4)(3)(2)(1)(1+x)^{-5}$

$f^{(6)}(x) = (-1)(5)(4)(3)(2)(1)(1+x)^{-6}$

Example 1:

Construct the Taylor series for $f(x) = \ln(1+x)$ centered at $a = 0$.

$f(x) = \ln(1+x)$ $\qquad$ $f^{(n)}(x) = (-1)^{n-1}(n-1)!(1+x)^{-n}$ for $n > 0$

$f'(x) = (1+x)^{-1}$

$f''(x) = (-1)(1+x)^{-2}$

$f^{(3)}(x) = (2)(1)(1+x)^{-3}$

$f^{(4)}(x) = (-1)(3)(2)(1)(1+x)^{-4}$

$f^{(5)}(x) = (4)(3)(2)(1)(1+x)^{-5}$

$f^{(6)}(x) = (-1)(5)(4)(3)(2)(1)(1+x)^{-6}$

Example 2:

Construct the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = 1$.

A hard integral

How to estimate $\int_0^{.5} e^{-x^2}\,dx$?

$$e^x = 1 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

$$e^{-x^2} = 1 - \frac{1}{1!}x^2 + \frac{1}{2!}x^4 - \frac{1}{3!}x^6 + \frac{1}{4!}x^8 + \cdots = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{n!}$$

$$\begin{aligned}
\int_0^{.5} e^{-x^2}\,dx &= \int_0^{.5} 1 - \frac{1}{1!}x^2 + \frac{1}{2!}x^4 - \frac{1}{3!}x^6 + \cdots\,dx \\
&= x - \frac{x^3}{1!\cdot 3} + \frac{x^5}{2!\cdot 5} - \frac{x^7}{3!\cdot 7} + \cdots \Bigg|_0^{.5} \\
&\approx (.5) - \frac{(.5)^3}{1!\cdot 3} + \frac{(.5)^5}{2!\cdot 5} - \frac{(.5)^7}{3!\cdot 7}
\end{aligned}$$