

Some notes from class

2018-04-10

Yesterday

If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$, then

- $f(x) = c_0 + c_1(x-a)^1 + c_2(x-a)^2 + c_3(x-a)^3 + \dots$
- $f'(x) = c_1 + 2c_2(x-a)^1 + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \dots$
- $f''(x) = 2c_2 + 3(2)c_3(x-a)^1 + 4(3)c_4(x-a)^2 + 5(4)c_5(x-a)^3 + \dots$
- $f^{(3)}(x) = (3)(2)(1)c_3 + (4)(3)(2)c_4(x-a)^1 + 5(4)(3)c_5(x-a)^2 + \dots$
- $f^{(4)}(x) = (4)(3)(2)(1)c_4 + 5(4)(3)(2)c_5(x-a) + \dots$

So

$$c_n = \frac{f^{(n)}(a)}{n!}$$

Taylor Series

If we begin with a function $f(x)$, then the series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$ is called the *Taylor series* for the function $f(x)$ centered at a .

Key Idea:

If $f(x)$ has a power series expansion at $x = a$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n.$$

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$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n.$$

Special case, $a = 0$ (called *Maclaurin series*):

If $f(x)$ has a power series expansion at $x = 0$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

Example 1:

Construct the Taylor series for $f(x) = e^x$ centered at $a = 1.6$.

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Answer:

$$e^{1.6} + \frac{e^{1.6}}{1!}(x - 1.6)^1 + \frac{e^{1.6}}{2!}(x - 1.6)^2 + \frac{e^{1.6}}{3!}(x - 1.6)^3 + \dots$$

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Construct the Maclaurin series for $f(x) = e^x$.

Answer:

$$e^0 + \frac{e^0}{1!}x^1 + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \dots$$

Example 2:

Construct the Maclaurin series for $f(x) = \sin x$ (i.e. Taylor series centered at $a = 0$).

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Start computing:

$$f^{(0)}(x) =$$

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$$f^{(8)}(x) =$$

$$f^{(1)}(x) =$$

$$f^{(5)}(x) =$$

$$f^{(9)}(x) =$$

$$f^{(2)}(x) =$$

$$f^{(6)}(x) =$$

$$f^{(10)}(x) =$$

$$f^{(3)}(x) =$$

$$f^{(7)}(x) =$$

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

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$$f^{(0)}(x) = \cos x$$

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$$f^{(8)}(x) = \cos x$$

$$f^{(1)}(x) = -\sin x$$

$$f^{(5)}(x) = -\sin x$$

$$f^{(9)}(x) = -\sin x$$

$$f^{(2)}(x) = -\cos x$$

$$f^{(6)}(x) = -\cos x$$

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$$f^{(9)}(x) = -\sin x$$

$$f^{(2)}(x) = -\cos x$$

$$f^{(6)}(x) = -\cos x$$

$$f^{(10)}(x) = -\cos x$$

$$f^{(3)}(x) = \sin x$$

$$f^{(7)}(x) = \sin x$$

$$f^{(11)}(x) = \sin x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$