

Some notes from class

2018-04-09

Things we know

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$
- $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$
- $\frac{1}{1+x^2} =$

If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n, \dots$

How do you find the coefficients c_0, c_1, c_2, \dots ?

- $f(x) = c_0 + c_1(x-a)^1 + c_2(x-a)^2 + c_3(x-a)^3 + \dots$
 $x = a \implies$

- $f'(x) = c_1 + 2c_2(x-a)^1 + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \dots$
 $x = a \implies$

- $f''(x) = 2c_2 + 3(2)c_3(x-a)^1 + 4(3)c_4(x-a)^2 + 5(4)c_5(x-a)^3 + \dots$
 $x = a \implies$

- $f^{(3)}(x) =$
 $x = a \implies$

Taylor Series

If we begin with a function $f(x)$, then the series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$ is called the *Taylor series* for the function $f(x)$ centered at a .

Key Idea:

If $f(x)$ has a power series expansion at $x = a$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n.$$

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Special case, $a = 0$ (called Maclaurin series):

If $f(x)$ has a power series expansion at $x = 0$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$