Some notes from class

2018-04-06

These are all “power series”

- $1 + x^1 + x^2 + x^3 + \cdots$
- $1 + \frac{1}{2}x^1 + \frac{2}{3}x^2 + \frac{3}{4}x^3 + \cdots$
- $0 + \frac{1}{2}(x - 2.5)^1 + \frac{2}{3}(x - 2.5)^2 + \frac{3}{4}(x - 2.5)^3 + \cdots$
- $2 + 5(x + 1) + 6(x + 1)^2 + 2(x + 1)^3 + 3(x + 1)^4 + 6(x + 1)^5 + \cdots$
- $1 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$
- $1 + \qquad \frac{-1}{2!}x^2 + \qquad \frac{1}{4!}x^4 + \qquad \frac{-1}{6!}x^6 + \cdots$

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- $2 + 5(x+1) + 6(x+1)^2 + 2(x+1)^3 + 3(x+1)^4 + 6(x+1)^5 + \cdots$
- $1 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$
- $1 + 0x^1 + \frac{-1}{2!}x^2 + 0x^3 + \frac{1}{4!}x^4 + 0x^5 + \frac{-1}{6!}x^6 + \cdots$

A *power series* is

Definition

A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$ is called a *power series* centered at $x = a$.

$$c_0 + c_1(x-a)^1 + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

Today: Can we take a given function and find a power series equal to that function?

Plan for this section

Take the (one and only) example we know of a function that is equal to a certain power series, and perform clever algebraic and calculus tricks on it.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad \text{for } |x| < 1$$

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For example: We can

- multiply a power series by a constant,
- multiply a power series by x,
- replace x with something else,
- add two power series (that both converge on a certain interval).

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Theorem (Key theoretical tool)

If the power series $\sum_n c_n(x-a)^n$ has a radius of convergence $R > 0$, then the function f defined by $f(x) = \sum_n c_n(x-a)^n$ is differentiable (and thus continuous) on the interval $(a-R, a+R)$, and

- $f'(x) =$ *exactly what you would expect,*
- $\int f(x)\,dx =$ *exactly what you would expect*

Note: Differentiation and antidifferentiation do not change the radius of convergence R.