Some notes from class

2018-04-04

These are all "power series"

- $1 + x^1 + x^2 + x^3 + \cdots$
- $1 + \frac{1}{2}x^1 + \frac{2}{3}x^2 + \frac{3}{4}x^3 + \cdots$
- $0 + \frac{1}{2}(x - 2.5)^1 + \frac{2}{3}(x - 2.5)^2 + \frac{3}{4}(x - 2.5)^3 + \cdots$
- $2 + 5(x + 1) + 6(x + 1)^2 + 2(x + 1)^3 + 3(x + 1)^4 + 6(x + 1)^5 + \cdots$
- $1 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$
- $1 + \qquad \frac{-1}{2!}x^2 + \qquad \frac{1}{4!}x^4 + \qquad \frac{-1}{6!}x^6 + \cdots$

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- $2 + 5(x + 1) + 6(x + 1)^2 + 2(x + 1)^3 + 3(x + 1)^4 + 6(x + 1)^5 + \cdots$
- $1 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$
- $1 + 0x^1 + \frac{-1}{2!}x^2 + 0x^3 + \frac{1}{4!}x^4 + 0x^5 + \frac{-1}{6!}x^6 + \cdots$

A *power series* is

Definition

A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$ is called a *power series* centered at $x = a$.

$$c_0 + c_1(x-a)^1 + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

Key Idea: We will think of a power series as a function.

Important Question: For which values of x does the series converge?

Example 1

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{3\sqrt{n}}$$

Example 2

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Example 3

$$\sum_{n=0}^{\infty} \frac{n!(x-5)^n}{3^n}$$

Where will my power series converge?

Theorem (Power Series Convergence)

If $\sum_{n=0}^{\infty} c_n(x-a)^n$ *is a power series, then one of the following three things happens (regarding convergence):*

- $\sum_{n=0}^{\infty} c_n(x-a)^n$ *converges only when* $x = a$*;*
- $\sum_{n=0}^{\infty} c_n(x-a)^n$ *converges for all real numbers* x
- $\sum_{n=0}^{\infty} c_n(x-a)^n$ *converges on an interval of the form* $(a-R, a+R)$ *or* $[a-R, a+R]$ *or* $[a-R, a+R)$ *or* $(a-R, a+R]$.

Example

Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-7)^n}{4^n n}$$

Example

Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{2^{n+1}(x-8)^n}{n^2 5^n}$$