Some notes from class

2018-04-02

Reminders

Geometric series: $a + ar + ar^2 + ar^3 + \cdots = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$

Integral Test: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} \quad \longleftrightarrow \quad \int_1^{\infty} \frac{1}{x^3}\,dx$

Comparison Test: Given $\sum_n a_n$, compare the terms of $\sum_n a_n$ to the terms of some other series you kow about.

Limit Comparison Test: Given $\sum_n a_n$, look at $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n}$, where $\sum_n b_n$ is some series you understand.

Alternating Series Test: Are the terms decreasing to 0, and are signs alternating?

Ratio Test: Look at $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$. $\quad L > 1,\ L < 1,\ L = 1$?

Root Test: Look at $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$. $\quad L > 1,\ L < 1,\ L = 1$?

The Root Test

Theorem (Root Test)

Suppose $\sum_n a_n$ *is a series.*

- *If* $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, *then the series* $\sum_n a_n$ *is absolutely convergent.*
- *If* $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, *then the series* $\sum_n a_n$ *is divergent.*
- *If* $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, *then the Root Test is inconclusive.*

Examples (How well does the Root Test work?)

- $\displaystyle\sum_{n=0}^{\infty} 5\left(\frac{3}{7}\right)^n$
- $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+5}{4n-1}\right)^n$
- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

Ratio and Root Tests (two more examples)

- $$\sum_{n=1}^{\infty} \frac{8n^2 5^{n+1}}{3^{2n}}$$

- $$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^n}$$

Assorted Examples

- $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{3^n + 4^n}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{5^n n^8}{n!}$
- $\displaystyle\sum_{k=3}^{\infty} \frac{1}{(\ln k)^{lnk}}$
- $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$
- $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-3n}$

Assorted Examples

- $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n^{5n}}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^6 + 5n^3}}{n^{4.1} + 7n}$
- $\displaystyle\sum_{k=1}^{\infty} \frac{\sin k}{1 + k^3}$
- $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$