Some notes from class

2018-03-27

Reminders

Geometric series:

$$a + ar + ar^2 + ar^3 + \cdots = \lim_{n\to\infty} \frac{a(1 - r^n)}{1 - r}$$

Harmonic series:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots = \text{diverges}$$

Integral Test: $$\sum_{n=1}^{\infty} \frac{n+5}{n^3+1} \quad \longleftrightarrow \quad \int_1^{\infty} \frac{x+5}{x^3+1}\,dx$$

Comparison Test: If $\sum_n b_n$ converges and $a_n \leq b_n$, then $\sum_n a_n$ converges.

Example

$$\sum_{k=1}^{\infty} \frac{1}{(\ln k)k^2}$$

Example

$$\sum_{n=1}^{\infty} \frac{3n^2 + 5n}{4n^5 - n^4}$$

Limit Comparison Test

Theorem (Limit Comparison Test (My favorite test))

Suppose $\sum_n a_n$ *and* $\sum_n b_n$ *are series with positive terms. If* $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, *where* c *is some nonzero positive (and finite) real number, then either both series converge or both series diverge.*

Examples

Each of these is well suited for the Limit Comparison Test.

- $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2 - 6n + 2}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n - 4}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4 - n}}{n^{3.5} + 5n^2}$

Getting ready for the Alternating Series Test

All of the following series have terms that approach 0. Only one of the series converges.

$$\sum_{n=1}^{\infty} \frac{1}{n} \qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad \sum_{n=1}^{\infty} \frac{1}{e^n} \qquad \sum_{n=3}^{\infty} \frac{1}{\ln(n)} \qquad \sum_{n=3}^{\infty} \frac{1}{n \ln(n)}$$

Alternating Series Test

Theorem (Alternating Series Test)

The series $\sum_{n=1}^{\infty}(-1)^n a_n$ *is guaranteed to converge if:*

- $a_n \geq 0$ *for all* n,
- $a_{n+1} \leq a_n$ *for all* n,
- $\lim_{n\to\infty} a_n = 0$.