Some notes from class

2018-03-26

Recall:

Geometric series

$$a + ar + ar^2 + ar^3 + \cdots = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$

Harmonic series

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots = \text{diverges}$$

Integral Test: If we want to know whether something like $\sum_{n=1}^{\infty} \frac{n+5}{n^3+1}$ converges, we can just check whether $\int_1^{\infty} \frac{x+5}{x^3+1}\,dx$ converges.

Example 1

Does $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{n^3+1}$ converge?

Example 2

Does $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k\ln(k)}$ converge?

Reasonable reasoning...

- If b is little and $a \leq b$, then a is little.
- If b is big and $a \geq b$, then a is big.

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Theorem

Suppose that the terms of the series $\sum a_n$ *and* $\sum b_n$ *are positive. Then:*

- *If* $\sum_{n=1}^{\infty} b_n$ *converges and* $a_n \leq b_n$ *(after a while), then* $\sum_{n=1}^{\infty} a_n$ *must also converge.*
- *If* $\sum_{n=1}^{\infty} b_n$ *diverges and* $a_n \geq b_n$ *(after a while), then* $\sum_{n=1}^{\infty} a_n$ *must also diverge.*