Some notes from class

2018-03-23

Recall:

Geometric series

$$a + ar + ar^2 + ar^3 + \cdots = \lim_{n\to\infty} \frac{a(1 - r^n)}{1 - r}$$

Harmonic series

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots = \text{diverges}$$

Recall:

$$\int_1^\infty \frac{1}{x}\,dx \text{ diverges,} \qquad \int_1^\infty \frac{1}{\sqrt{x}}\,dx \text{ diverges,} \qquad \int_1^\infty \frac{1}{x^2}\,dx \text{ converges}$$

The Integral Test

Suppose $f(x)$ is a continuous, positive, decreasing function on $[1, \infty)$. Then the convergence of $\int_1^\infty f(x), dx$ is equivalent to the convergence of the series $\sum_{n=1}^{\infty} a_n$, where $a_n = f(n)$.

In other words:

1. If $\int_1^\infty f(x), dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\int_1^\infty f(x), dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.