Some notes from class

2018-03-21

Overview of Series

Goal: Find value of $a_1 + a_2 + a_3 + \cdots = \sum_{i=1}^{\infty} a_i$

Method: Form "sequence of partial sums." $\quad s_n = a_1 + a_2 + \cdots + a_n.$
Take limit: $\quad \lim_{n\to\infty} s_n \quad$ (Often hard to find a formula for s_n.)

Example: **(Geometric Series)** $a + ar + ar^2 + ar^3 + \cdots$

$$S = a + ar + ar^2 + \cdots + ar^{n-1} \qquad n\text{th partial sum}$$

$$rS = \qquad ar + ar^2 + \cdots + ar^{n-1} + ar^n$$

$$S - rS = a - ar^n$$

$$S = \frac{a(1-r^n)}{1-r} \qquad n\text{th partial sum}$$

Geometric series

$$a + ar + ar^2 + ar^3 + \cdots = \lim_{n\to\infty} \frac{a(1 - r^n)}{1 - r}$$

Example 1: $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots$

Example 2: $6 + 6\left(\frac{4}{3}\right) + 6\left(\frac{4}{3}\right)^2 + 6\left(\frac{4}{3}\right)^3 + \cdots$

Harmonic series

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{16} + \cdots$$

Harmonic series

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\cdots+\frac{1}{16}+\cdots$$

$$=\frac{1}{1}+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\left(\frac{1}{9}+\frac{1}{10}+\cdots+\frac{1}{16}\right)+\cdots$$

True or false?

Statement 1. True or false?

If $\sum_{m=1}^{\infty} a_m$ converges, then $\lim_{m \to \infty} a_m = 0$.

Statement 2. True or false?

If $\lim_{m \to \infty} a_m = 0$, then $\sum_{m=1}^{\infty} a_m$ converges.