Some notes from class

2018-02-28

$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for parametric formulas

If x and y depend on t, then

$$\frac{dy}{dx} = \text{rate of change of } y \text{ relative to } x = \frac{\frac{d}{dt}(y)}{\frac{d}{dt}(x)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^x} = \text{rate of change of } \tfrac{dy}{dx} \text{ relative to } x = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{d}{dt}(x)} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Example: $x = t^2 + 1, \quad y = t^3 - 3t \implies \frac{dy}{dx} = \frac{3t^2-3}{2t} = \frac{3}{2}t - \frac{3}{2}t^{-1}$
Thus

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Thus

$$\frac{d^2y}{dx^2} = \frac{\frac{3}{2} + \frac{3}{2}t^{-2}}{2t} = \frac{3(t^2+1)}{4t^3}$$

Three concepts from earlier

$$\text{Area} = \int_a^b f(x)\,dx$$

$$\text{Are Length} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$\text{Surface Area} = \int_a^b 2\pi \underbrace{f(x)}_{\text{radius}} \underbrace{\sqrt{1 + (f'(x))^2}\,dx}_{\text{arclength}}$$

Area

$$\text{Area} = \int_a^b \underbrace{f(x)}_{\text{height}}\, dx = \int_\alpha^\beta \underbrace{y(t)}_{\text{height}}\, \underbrace{x'(t)\, dt}_{dx}$$

Example: $x = 4 - t^2 \qquad y = 16 - 8t^2 + t^4$