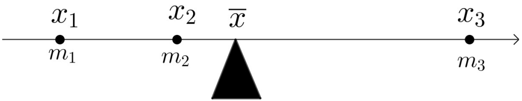
Some notes from class

2018-02-20

Center of Mass - Very Easy Example

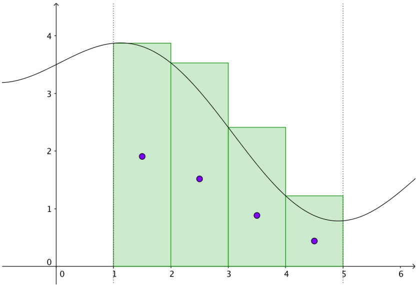


Balances if: $m_1(\overline{x} - x_1) + m_2(\overline{x} - x_2) = m_3(x_3 - \overline{x})$

$$\implies (m_1 + m_2 + m_3)\overline{x} = m_1x_1 + m_2x_2 + m_3x_3$$

$$\implies \overline{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

How to find center of mass $(\overline{x}, \overline{y})$ of a given shape

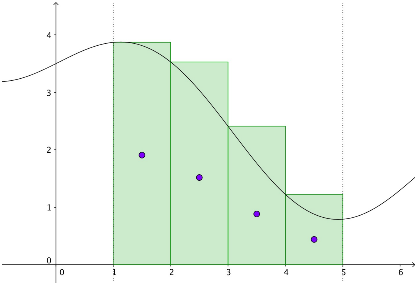


$$\overline{x} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_k x_k}{m_1 + m_2 + \cdots + m_k} = \frac{M_y}{M}$$

$$\overline{y} = \frac{m_1 y_1 + m_2 y_2 + \cdots + m_k y_k}{m_1 + m_2 + \cdots + m_k} = \frac{M_x}{M}$$

1. We assume the shape has density of ρ kg/m^2.
2. If shape is rectangular, the center of mass is in the obvious place.
3. To find center of mass of large object, break it into many rectangles, & treat each rectangle as a point mass.

How to find center of mass $(\overline{x}, \overline{y})$ of a given shape



$$\overline{x} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_k x_k}{m_1 + m_2 + \cdots + m_k} = \frac{M_y}{M}$$

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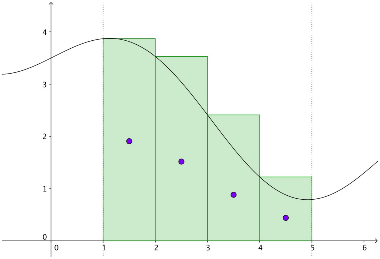
How to find center of mass $(\overline{x}, \overline{y})$ of a given shape



1. $M = \int_a^b \rho f(x)\, dx$
2. $M_y = \int_a^b \rho\, x f(x)\, dx$
3. $M_x = \frac{\rho}{2} \int_a^b f(x)^2\, dx$

$$\overline{x} = \frac{M_y}{M} \qquad \overline{y} = \frac{M_x}{M}$$

Think of

$$\overline{y} = \text{Sum of terms of form } \underbrace{\text{vertical position}}_{\frac{f(x)}{2}} \times \underbrace{\text{mass}}_{\rho f(x)\, dx}$$

Trying to model $M_x = y_1 m_1 + y_2 m_2 + \cdots + y_k m_k$.

Center of mass of upper half circle $f(x) = \sqrt{1-x^2}$

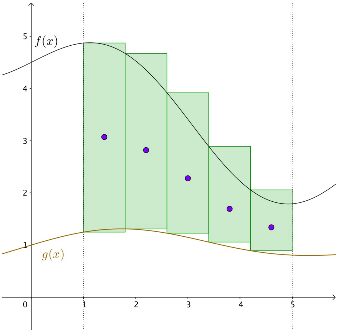
$$M = \int_a^b \rho f(x)\,dx, \qquad M_y = \int_a^b \rho\, x f(x)\,dx, \qquad M_x = \frac{\rho}{2}\int_a^b f(x)^2\,dx$$

$\overline{x} = 0$

To find $\overline{y}$, $\quad M_x = \frac{\rho}{2}\int_{-1}^{1}(\sqrt{1-x^2})^2\,dx = \frac{\rho}{2}\int_{-1}^{1} 1 - x^2\,dx = \cdots = \frac{2\rho}{3}$

$$\overline{y} = \left(\frac{2\rho}{3}\right) / \left(\frac{\pi\rho}{2}\right) = \frac{4}{3\pi} \approx 0.4244\ldots$$

How to find C.O.M. $(\overline{x}, \overline{y})$ of shape bounded between. . .



$$\overline{x} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_k x_k}{m_1 + m_2 + \cdots + m_k} = \frac{M_y}{M}$$

$$\overline{y} = \frac{m_1 y_1 + m_2 y_2 + \cdots + m_k y_k}{m_1 + m_2 + \cdots + m_k} = \frac{M_x}{M}$$