Some notes from class

2018-02-12

Reminder of estimates for $\int_a^b f(x)\,dx$

$$M_8 = f\left(\frac{x_0 + x_1}{2}\right)\Delta x + f\left(\frac{x_1 + x_2}{2}\right)\Delta x + \cdots + f\left(\frac{x_7 + x_8}{2}\right)\Delta x$$

$$T_8 = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_7) + f(x_8)\right)$$

$$S_8 = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + y_8\right)$$

How much work to fly a rocket into space?

Work = Force $\times$ Distance

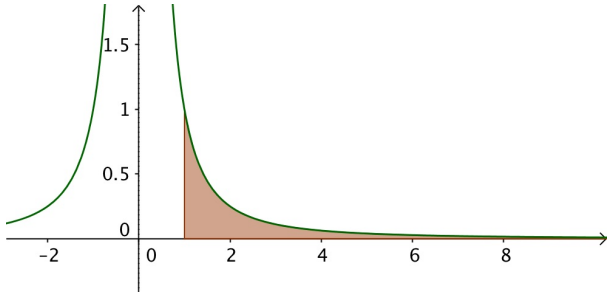
$F(x) = \frac{Gm_1m_2}{x^2}$ force at position x meters from center of earth

Work to move from position x to position $x + \Delta x$ is $\approx F(x)\Delta x$

$$\text{Work (from position } a \text{ to } b) = C\int_a^b \frac{1}{x^2}\,dx \qquad \text{where } C = Gm_1m_2$$

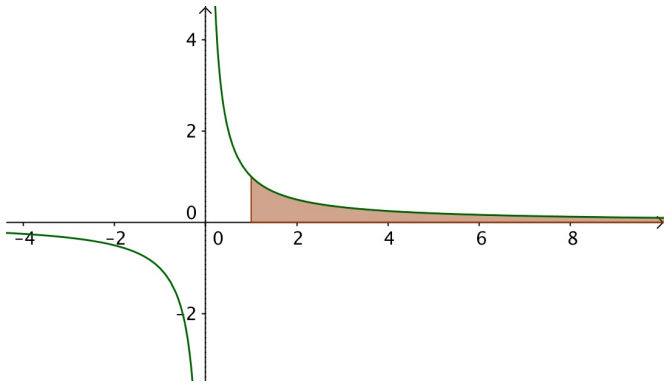
What's about to happen? (Version 1)

$$\int_1^\infty \frac{1}{x^2}\,dx$$



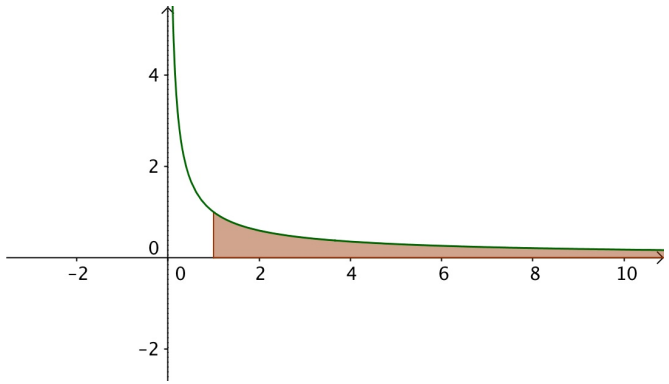
finite (converges)

$$\int_1^\infty \frac{1}{x^1}\,dx$$



not finite (diverges)

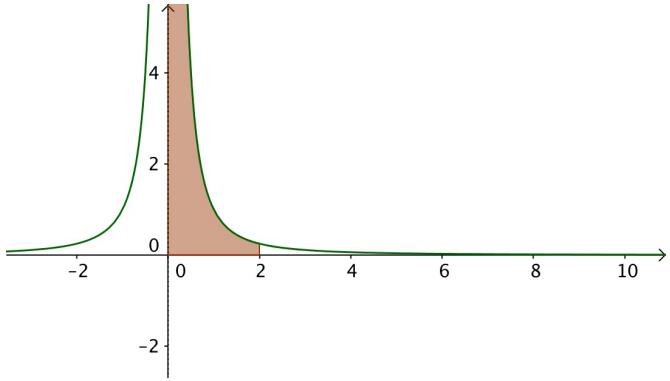
$$\int_1^\infty \frac{1}{x^{3/4}}\,dx$$



not finite (diverges)

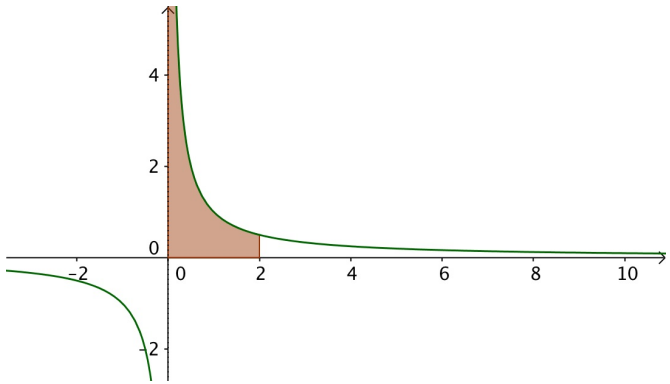
What's about to happen? (Version 2)

$$\int_0^2 \frac{1}{x^2}\,dx$$



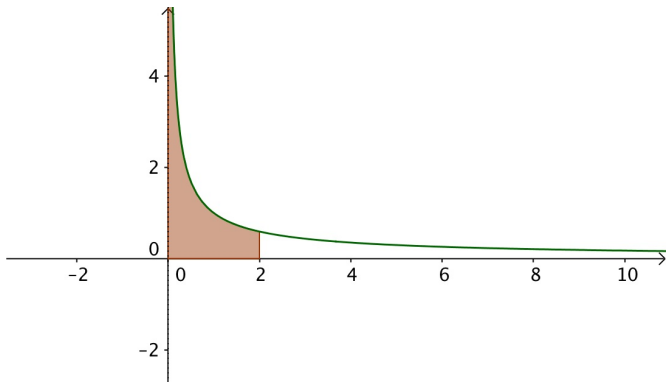
not finite (diverges)

$$\int_0^2 \frac{1}{x^1}\,dx$$



not finite (diverges)

$$\int_0^2 \frac{1}{x^{3/4}}\,dx$$



finite (converges)

Some conceptual questions

1. True/False: If the domain of $f(x)$ is all real numbers and $\int_{25}^{\infty} f(x)\,dx$ converges, then $\int_{1}^{\infty} f(x)\,dx$ converges.

2. True/False: $\int_{-3}^{3} \frac{1}{x}\,dx = \ln|3| - \ln|-3| = 0$

3. True/False: Recall that we think of $\frac{1}{x}$ as something of a tipping point. If $f(x) < \frac{1}{x}$ for all x, then $\int_{3}^{\infty} f(x)\,dx$ converges.

4. True/False: If $\int_{3}^{\infty} f(x)\,dx$ diverges, then $\int_{25}^{\infty} f(x)\,dx$ diverges.