Some notes from class

2018-01-31

Partial fraction decomposition

Four cases for how to decompose $\frac{p(x)}{q(x)}$, where $\deg p < \deg q$.

1. $$\int \frac{p(x)}{(x-2)(x+3)(x-7)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x-7}\,dx$$

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2. $$\int \frac{p(x)}{(x-1)^3(x+6)(x-7)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \\ + \frac{D}{x+6} + \frac{E}{X-7}\,dx$$

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3. $$\int \frac{p(x)}{(x^2+9)(x^2+x+4)(x-3)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{Cx+D}{x^2+x+4} + \frac{E}{x-3}\,dx$$

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4. $$\int \frac{p(x)}{(x^2+9)^2(x+5)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{Cx+D}{(x^2+9)^2} + \frac{E}{x+5}\,dx$$

Examples of partial fraction decomposition

1. $$\int \frac{x-11}{(x-1)(x+4)}\,dx = \int \frac{3}{x+4} - \frac{2}{x-1}\,dx$$
2. $$\int \frac{5x^3-13x^2+11}{(x-1)^3(x+2)}\,dx = \int \frac{2}{x-1} - \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3} + \frac{3}{x+2}\,dx$$
3. $$\int \frac{-x^2-9x-32}{(x-5)(x^2+9)}\,dx = \int \frac{2x+1}{x^2+9} - \frac{3}{x-5}\,dx$$
4. $$\int \frac{5x^4+x^3+x^2+3x+5}{(x+2)(x^2+1)^2}\,dx = \int \frac{2x-3}{x^2+1} + \frac{-x+4}{(x^2+1)^2} + \frac{3}{x+2}\,dx$$

Reminder. You can only do this to $\frac{p(x)}{q(x)}$ if $\deg p < \deg q$.

Examples of partial fraction decomposition

Yesterday, we did this:

$$\frac{x-11}{(x-1)(x+4)} = \frac{A}{x+4} + \frac{B}{x-1} = \frac{A(x-1)+B(x+4)}{(x+4)(x-1)}$$

$$\implies x - 11 = A(x-1) + B(x+4)$$

$$x = 1 \implies B = -2$$

$$x = -4 \implies A = 3$$

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$$x = -4 \implies A = 3$$

Today:

$$1x + (-11) = (A+B)x + (-A+4B)$$

$$\begin{aligned} A + B &= 1 \\ -A + 4B &= -11 \end{aligned}$$

The big picture

How to compute $\displaystyle\int \frac{x^4+6x^3+5x^2-11x-11}{(x-1)(x+4)}\,dx$

$$\frac{x^4+6x^3+5x^2-11x-11}{(x-1)(x+4)} = \underbrace{x^2+3x}_{\text{quotient}} + \frac{\overbrace{x-11}^{\text{remainder}}}{(x-1)(x+4)}$$

$$\int \frac{x^4+6x^3+5x^2-11x-11}{(x-1)(x+4)}\,dx = \int x^2+3x+\frac{x-11}{(x-1)(x+4)}\,dx$$

$$\int \frac{x-11}{(x-1)(x+4)}\,dx = \int \frac{3}{x+4}-\frac{2}{x-1}\,dx$$

Another example of partial fraction decomposition

$$\int \frac{-x^2-9x-32}{(x-5)(x^2+9)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{C}{x-5}\,dx$$

$$\frac{Ax+B}{x^2+9} + \frac{C}{x-5} = \frac{(Ax+B)(x-5)+C(x^2+9)}{(x^2+9)(x-5)}$$

$$-x^2-9x-32 \quad = \quad (Ax+B)(x-5)+C(x^2+9)$$

Another example of partial fraction decomposition

$$\int \frac{-x^2-9x-32}{(x-5)(x^2+9)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{C}{x-5}\,dx$$

$$\frac{Ax+B}{x^2+9} + \frac{C}{x-5} = \frac{(Ax+B)(x-5)+C(x^2+9)}{(x^2+9)(x-5)}$$

$$\begin{aligned} -x^2-9x-32 &= (Ax+B)(x-5)+C(x^2+9) \\ &= (A+C)x^2+(-5A+B)x+(-5B+9C) \end{aligned}$$

$$\begin{aligned} A \quad + \quad C &= -1 \\ -5A+B &= -9 \\ -5B+9C &= -32 \end{aligned}$$

Don't forget the big picture

Two very different examples that look similar.

$$\int \frac{8x-7}{x^2-x-2}\,dx$$

$$\int \frac{3x+4}{x^2-6x+25}\,dx$$

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$$\int \frac{8x-7}{x^2-x-2}\,dx \quad = \int \frac{8x-7}{(x-2)(x+1)}\,dx$$

$$\int \frac{3x+4}{x^2-6x+25}\,dx \quad = \int \frac{3x+4}{(x-3)^2+16}\,dx$$

Don't forget the big picture

Two very different examples that look similar.

$$\int \frac{8x-7}{x^2-x-2}\,dx = \int \frac{8x-7}{(x-2)(x+1)}\,dx = \int \frac{3}{x-2}+\frac{5}{x+1}\,dx$$

$$\int \frac{3x+4}{x^2-6x+25}\,dx = \int \frac{3x+4}{(x-3)^2+16}\,dx = \int \frac{3(u+3)+4}{u^2+16}\,dx$$