Some notes from class

2018-01-30

$\int \frac{1}{\sqrt{x^2+4x+8}}\,dx$

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta \qquad u = 2\tan\theta \\
&= \int \sec\theta\,d\theta \\
&= \ln|\sec\theta+\tan\theta| + C \\
&= \ln\left|\sec\left(\tan^{-1}\left(\frac{u}{2}\right)\right)+\frac{u}{2}\right| + C
\end{aligned}$$

$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx$

$$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx = \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx$$

$$= \int \frac{u^2+2u+2}{(u^2+1)^2}\,du$$

$$= \int \frac{u^2+1}{(u^2+1)^2}\,du + \int \frac{2u}{(u^2+1)^2}\,du + \int \frac{1}{(u^2+1)^2}\,du$$

$$= \int \frac{1}{u^2+1}\,du + \underset{\text{let } w = u^2+1}{\text{easy}} + \underset{\substack{\text{let } u = \tan\theta \\ du = \sec^2\theta\,d\theta}}{\text{yuck}}$$

$$= \int \frac{1}{u^2+1}\,du + \int \frac{1}{w^2}\,dw + \int \frac{\sec^2\theta}{(\tan^2\theta+1)^2}\,d\theta$$

$$= \tan^{-1} u - \frac{1}{u^2+1} + \int \frac{1}{\sec^2\theta}\,d\theta$$

Here are some things we can do so far

1. $\displaystyle\int \frac{1}{x-3}\,dx$ Easy
2. $\displaystyle\int \frac{1}{(x-3)^5}\,dx$ Easy
3. $\displaystyle\int \frac{1}{1+x^2}\,dx$ Easy
4. $\displaystyle\int \frac{3+6x}{9+x^2}\,dx$ Not too bad
5. $\displaystyle\int \frac{2-10x}{(9+x^2)^2}\,dx$ Bad, but possible
6. $\displaystyle\int \frac{1}{x^2-8x+25}\,dx$ Not bad because $\displaystyle = \int \frac{1}{(x-4)^2+9}\,dx$
7. $\displaystyle\int \frac{1}{x^2-10x+21}\,dx = \int \frac{1}{(x-5)^2-4}\,dx$ Now what?

Section 7.4 Overview

Plan: Turn every integral of the form $\int \frac{\text{polynomial}}{\text{polynomial}}\,dx$ into one or more of the integrals on the previous page.

1. $$\int \frac{x-11}{(x-1)(x+4)}\,dx = \int \frac{3}{x+4} - \frac{2}{x-1}\,dx$$

2. $$\int \frac{5x^3-13x^2+11}{(x-1)^3(x+2)}\,dx = \int \frac{2}{x-1} - \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3} + \frac{3}{x+2}\,dx$$

3. $$\int \frac{-x^2-9x-32}{(x-5)(x^2+9)}\,dx = \int \frac{2x+1}{x^2+9} - \frac{3}{x-5}\,dx$$

4. $$\int \frac{5x^4+x^3+x^2+3x+5}{(x+2)(x^2+1)^2}\,dx = \int \frac{2x-3}{x^2+1} + \frac{-x+4}{(x^2+1)^2} + \frac{3}{x+2}\,dx$$

Partial fraction decomposition

Four cases for how to decompose $\frac{p(x)}{q(x)}$, where $\deg p < \deg q$.

1. $$\int \frac{p(x)}{(x-2)(x+3)(x-7)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x-7}\,dx$$

Partial fraction decomposition

Four cases for how to decompose $\frac{p(x)}{q(x)}$, where $\deg p < \deg q$.

1. $$\int \frac{p(x)}{(x-2)(x+3)(x-7)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x-7}\,dx$$

2. $$\int \frac{p(x)}{(x-1)^3(x+6)(x-7)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+6} + \frac{E}{X-7}\,dx$$

Partial fraction decomposition

Four cases for how to decompose $\frac{p(x)}{q(x)}$, where $\deg p < \deg q$.

1. $$\int \frac{p(x)}{(x-2)(x+3)(x-7)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x-7}\,dx$$

2. $$\int \frac{p(x)}{(x-1)^3(x+6)(x-7)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+6} + \frac{E}{X-7}\,dx$$

3. $$\int \frac{p(x)}{(x^2+9)(x^2+x+4)(x-3)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{Cx+D}{x^2+x+4} + \frac{E}{x-3}\,dx$$

Partial fraction decomposition

Four cases for how to decompose $\frac{p(x)}{q(x)}$, where $\deg p < \deg q$.

1. $$\int \frac{p(x)}{(x-2)(x+3)(x-7)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x-7}\,dx$$

2. $$\int \frac{p(x)}{(x-1)^3(x+6)(x-7)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+6} + \frac{E}{X-7}\,dx$$

3. $$\int \frac{p(x)}{(x^2+9)(x^2+x+4)(x-3)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{Cx+D}{x^2+x+4} + \frac{E}{x-3}\,dx$$

4. $$\int \frac{p(x)}{(x^2+9)^2(x+5)}\,dx = \int \frac{Ax+B}{x^2+9} + \frac{Cx+D}{(x^2+9)^2} + \frac{E}{x+5}\,dx$$