Some notes from class

2018-01-29

$$\int \frac{\sqrt{9x^2-4}}{x}\,dx \quad = \quad 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta$$

$$\int \frac{\sqrt{9x^2-4}}{x}\,dx = 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta$$

$$= 3\int \frac{\sqrt{\frac{4}{9}(\sec^2\theta - 1)}}{\frac{2}{3}\sec\theta}\,\frac{2}{3}\sec\theta\tan\theta\,d\theta$$

$$\begin{aligned}
\int \frac{\sqrt{9x^2-4}}{x}\,dx &= 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta \\
&= 3\int \frac{\sqrt{\frac{4}{9}(\sec^2\theta - 1)}}{\frac{2}{3}\sec\theta}\,\frac{2}{3}\sec\theta\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}(\sec^2\theta - 1)}\,\tan\theta\,d\theta
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sqrt{9x^2-4}}{x}\,dx &= 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta \\
&= 3\int \frac{\sqrt{\frac{4}{9}(\sec^2\theta - 1)}}{\frac{2}{3}\sec\theta}\,\frac{2}{3}\sec\theta\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}(\sec^2\theta - 1)}\,\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}\tan^2\theta}\,\tan\theta\,d\theta
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sqrt{9x^2-4}}{x}\,dx &= 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta \\
&= 3\int \frac{\sqrt{\frac{4}{9}(\sec^2\theta - 1)}}{\frac{2}{3}\sec\theta}\,\frac{2}{3}\sec\theta\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}(\sec^2\theta - 1)}\,\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}\tan^2\theta}\,\tan\theta\,d\theta \\
&= 2\int \tan^2\theta\,d\theta = 2\int (\sec^2\theta - 1)\,d\theta
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sqrt{9x^2-4}}{x}\,dx &= 3\int \frac{\sqrt{x^2-4/9}}{x}\,dx \qquad \text{Now let } x = \tfrac{2}{3}\sec\theta \\
&= 3\int \frac{\sqrt{\frac{4}{9}(\sec^2\theta - 1)}}{\frac{2}{3}\sec\theta}\,\frac{2}{3}\sec\theta\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}(\sec^2\theta-1)}\,\tan\theta\,d\theta \\
&= 3\int \sqrt{\frac{4}{9}\tan^2\theta}\,\tan\theta\,d\theta \\
&= 2\int \tan^2\theta\,d\theta = 2\int(\sec^2\theta - 1)\,d\theta \\
&= 2(\qquad\qquad\qquad\qquad) + C
\end{aligned}$$

$$\int \frac{1}{\sqrt{x^2+4x+8}}\,dx \quad = \quad \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx$$

$$\begin{aligned}\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\ &= \int \frac{1}{\sqrt{u^2+4}}\,du\end{aligned}$$

$$\begin{aligned}\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\ &= \int \frac{1}{\sqrt{u^2+4}}\,du \\ &= \int \frac{1}{\sqrt{u^2+4}}\,du\end{aligned}$$

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta
\end{aligned}$$

$$
\begin{aligned}
\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta \\
&= \int \sec\theta\,d\theta
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{1}{\sqrt{x^2+4x+8}}\,dx &= \int \frac{1}{\sqrt{(x+2)^2+4}}\,dx \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{u^2+4}}\,du \\
&= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta \\
&= \int \sec\theta\,d\theta \\
&= \ln\left|\sec\theta+\tan\theta\right|+C \\
&=
\end{aligned}
$$

$$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx \quad = \quad \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx$$

$$\begin{aligned}\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx &= \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx \\ &= \int \frac{u^2+2u+2}{(u^2+1)^2}\,du \\ &= \int \frac{u^2+1+2u+1}{(u^2+1)^2}\,du\end{aligned}$$

$$\begin{aligned}\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx &= \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx \\ &= \int \frac{u^2+2u+2}{(u^2+1)^2}\,du \\ &= \int \frac{u^2+1+2u+1}{(u^2+1)^2}\,du\end{aligned}$$

$$= \int \frac{u^2+1}{(u^2+1)^2}\,du + \int \frac{2u}{(u^2+1)^2}\,du + \int \frac{1}{(u^2+1)^2}\,du$$