Some notes from class

2018-01-26

$\int \cos^6 x \sin^2 x \, dx$

$$\begin{aligned}
\int \cos^6 x \sin^2 x \, dx &= \int (\cos^2 x)^3 \sin^2 x \, dx \\
&= \int \left(\frac{1+\cos(2x)}{2}\right)^3 \left(\frac{1-\cos(2x)}{2}\right) dx \\
&= \frac{1}{16} \int 1 + 2\cos(2x) - 2\cos^3(2x) - \cos^4(2x) \, dx
\end{aligned}$$

$$\int \cos^6 x \sin^2 x \, dx = \frac{1}{16}\left(x + \sin(2x) - 2\int \cos^3(2x)\, dx - \int \cos^4(2x)\, dx\right)$$

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$u = 2x \qquad du = 2\, dx$

$$\begin{aligned}
\int \cos^4(2x)\, dx &= \frac{1}{2}\int \cos^4 u \, du = \frac{1}{2}\int \left(\frac{1+\cos(2u)}{2}\right)^2 du \\
&= \frac{1}{8}\int 1 + 2\cos(2u) + \cos^2(2u)\, du \\
&= \frac{1}{8}\left(u + \sin(2u) + \int \cos^2(2u)\, du\right) \\
&= \frac{1}{8}\left(u + \sin(2u) + \int \underbrace{\frac{1+\cos(4u)}{2}}_{\text{sort of easy}}\, du\right)
\end{aligned}$$

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$$\int \cos^6 x \sin^2 x \, dx = \frac{1}{16}\left(x + \sin(2x) - 2\int \cos^3(2x)\, dx - \int \cos^4(2x)\, dx\right)$$

$u = 2x \qquad du = 2\, dx$

$$\begin{aligned} \int \cos^3(2x)\, dx &= \frac{1}{2}\int \cos^3 u \, du = \frac{1}{2}\int (1-\sin^2 u)\cos u \, du \\ &= \frac{1}{2}\int (1-w^2)\, dw \qquad \text{where } w = \sin u \\ &= \frac{1}{2}\left(w - \frac{w^3}{3}\right) \\ &= \frac{1}{2}\left(\sin(2x) - \frac{\sin^3(2x)}{3}\right) \end{aligned}$$

$$\text{Integral} = \frac{1}{16}\left(x + \sin(2x) - \sin(2x) + \frac{\sin^3(2x)}{3} - \int \cos^4(2x)\, dx\right)$$