Some notes from class

2018-01-24

Trig Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Note

$$\sin^2 0 = \frac{1 - \cos(2(0))}{2} \qquad \cos^2 0 = \frac{1 + \cos(2(0))}{2}$$

$$\int \sin^2 x \, dx = \int \frac{1 - \cos(2x)}{2} \, dx = \frac{1}{2} \int 1 - \cos(2x) \, dx$$
$$= \frac{1}{2} \left(x - \frac{\sin(2x)}{2} \right) + C$$

Example: Integration by parts

$$
\begin{aligned}
\int \sin^2 x \, dx &= \int \underbrace{(\sin x)}_{f} \underbrace{(\sin x)}_{g'} \, dx \\
&= \underbrace{(\sin x)}_{f} \underbrace{(-\cos x)}_{g} - \int \underbrace{(-\cos x)}_{g} \underbrace{(\cos x)}_{f'} \, dx \\
&= -\sin x \cos x + \int \cos^2 x \, dx
\end{aligned}
$$

Example: Integration by parts

$$
\begin{aligned}
\int \sin^2 x \, dx &= \int \underbrace{(\sin x)}_{f} \underbrace{(\sin x)}_{g'} \, dx \\
&= \underbrace{(\sin x)}_{f} \underbrace{(-\cos x)}_{g} - \int \underbrace{(-\cos x)}_{g} \underbrace{(\cos x)}_{f'} \, dx \\
&= -\sin x \cos x + \int \cos^2 x \, dx \\
&= -\sin x \cos x + \int 1 - \sin^2 x \, dx
\end{aligned}
$$

Example: Integration by parts

$$
\begin{aligned}
\int \sin^2 x \, dx &= \int \underbrace{(\sin x)}_{f} \underbrace{(\sin x)}_{g'} \, dx \\
&= \underbrace{(\sin x)}_{f} \underbrace{(-\cos x)}_{g} - \int \underbrace{(-\cos x)}_{g} \underbrace{(\cos x)}_{f'} \, dx \\
&= -\sin x \cos x + \int \cos^2 x \, dx \\
&= -\sin x \cos x + \int 1 - \sin^2 x \, dx \\
\color{blue}{\int \sin^2 x \, dx} &= -\sin x \cos x + x - \color{blue}{\int \sin^2 x \, dx}
\end{aligned}
$$

Example: Integration by parts

$$\begin{aligned}\int x^2 e^x \, dx &= \int \underbrace{x^2}_{f} \underbrace{e^x}_{g'} \, dx \\ &= x^2 e^x - \int 2x e^x \, dx\end{aligned}$$

Example: Integration by parts

$$\begin{aligned}\int x^2 e^x \, dx &= \int \underbrace{x^2}_{f} \underbrace{e^x}_{g'} \, dx \\ &= x^2 e^x - \int 2x e^x \, dx \\ &= x^2 e^x - 2 \int \underbrace{x}_{f} \underbrace{e^x}_{g'} \, dx\end{aligned}$$

$\int \cos^6 x \sin^2 x \, dx$

$$\begin{aligned}
\int \cos^6 x \sin^2 x \, dx &= \int (\cos^2 x)^3 \sin^2 x \, dx \\
&= \int \left(\frac{1+\cos(2x)}{2}\right)^3 \left(\frac{1-\cos(2x)}{2}\right) dx \\
&= \frac{1}{16} \int 1 + 2\cos(2x) - 2\cos^3(2x) - \cos^4(2x) \, dx
\end{aligned}$$

$\int \cos^6 x \sin^2 x \, dx$

$$\begin{aligned}
\int \cos^6 x \sin^2 x \, dx &= \int (\cos^2 x)^3 \sin^2 x \, dx \\
&= \int \left(\frac{1+\cos(2x)}{2}\right)^3 \left(\frac{1-\cos(2x)}{2}\right) dx \\
&= \frac{1}{16} \int 1 + 2\cos(2x) - 2\cos^3(2x) - \cos^4(2x) \, dx
\end{aligned}$$

$$\int \cos^6 x \sin^2 x \, dx = \frac{1}{16}\left(x + \sin(2x) - 2\int \cos^3(2x) \, dx - \int \cos^4(2x) \, dx\right)$$

$\int \cos^6 x \sin^2 x \, dx$

$$\begin{aligned} \int \cos^6 x \sin^2 x \, dx &= \int (\cos^2 x)^3 \sin^2 x \, dx \\ &= \int \left(\frac{1+\cos(2x)}{2}\right)^3 \left(\frac{1-\cos(2x)}{2}\right) dx \\ &= \frac{1}{16} \int 1 + 2\cos(2x) - 2\cos^3(2x) - \cos^4(2x) \, dx \end{aligned}$$

$$\int \cos^6 x \sin^2 x \, dx = \frac{1}{16}\left(x + \sin(2x) - 2\int \cos^3(2x)\, dx - \int \cos^4(2x)\, dx\right)$$

Note: $\int \cos^4(2x)\, dx = \frac{1}{2}\int \cos^4 u \, du = \frac{1}{2}\int \left(\frac{1+\cos(2u)}{2}\right)^2 du$

$u = 2x \qquad du = 2\, dx$