Some notes from class

2018-01-19

Derivative of $\sin^{-1} x$

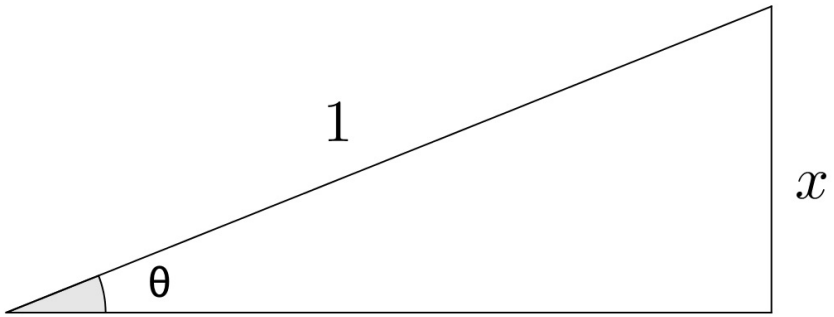
$f(x) = \sin x \qquad f^{-1}(x) = \sin^{-1} x$

$f'(x) = \cos x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x)} = \frac{1}{\cos(\sin^{-1} x)} = \frac{1}{\cos(\theta)}$$

$\theta = \sin^{-1} x$

$\sin\theta = x$



Derivative of $\tan^{-1} x$

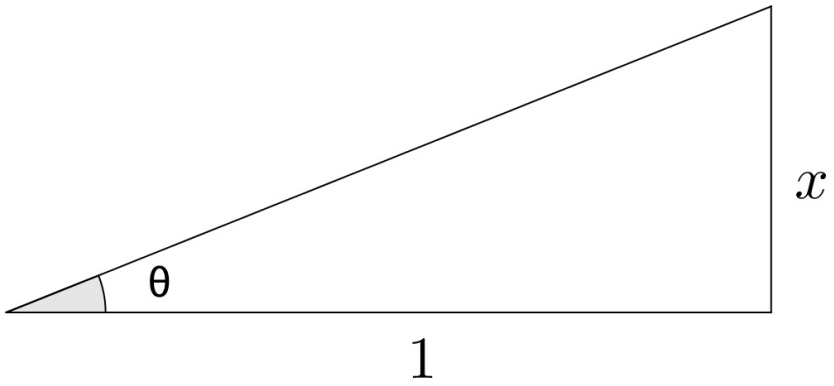
$f(x) = \tan x \qquad f^{-1}(x) = \tan^{-1} x$

$f'(x) = \sec^2 x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x)} = \frac{1}{\sec^2(\tan^{-1} x)} = \frac{1}{\sec^2(\theta)}$$

$\theta = \tan^{-1} x$

$\tan\theta = x$



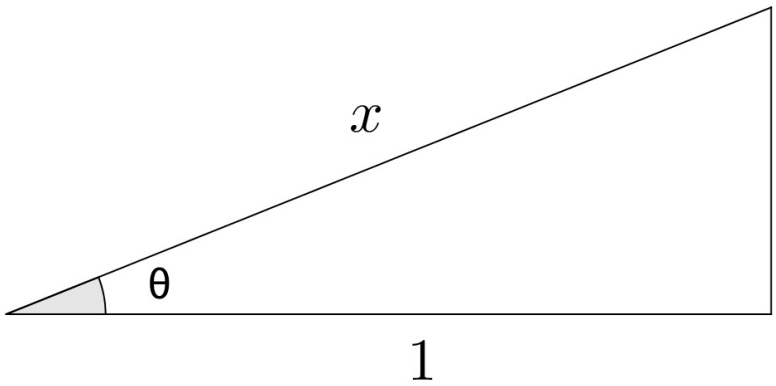
Derivative of $\sec^{-1} x$

$f(x) = \sec x \qquad f^{-1}(x) = \sec^{-1} x$

$f'(x) = \sec x \tan x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x)} = \frac{1}{\sec(\sec^{-1} x)\tan(\sec^{-1} x)} = \frac{1}{x\tan(\theta)}$$

$\theta = \sec^{-1} x, \qquad \sec\theta = x$



Summary

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

Make sure to memorize the blue formulas.

Some derivatives

$f(x) = \tan^{-1}(3x + x^5)$ $\qquad$ $f'(x) =$

$g(x) = \sin^{-1}((\ln x)\cos x)$ $\qquad$ $g'(x) =$

Some antiderivatives

$$\int \frac{5}{1+x^2}\,dx$$

Some antiderivatives

$$\int \frac{5}{1+(3x+4)^2}\,dx$$

Some antiderivatives

$$\int \frac{1}{\sqrt{1-(2x+5)^2}}\,dx$$

Some antiderivatives

$$\int \frac{1}{x^2+9}\,dx$$

Some antiderivatives

$$\int \frac{1}{x^2 + 6x + 10}\, dx$$