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Bibliographic Information

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Call Number:

Title: Experimental psychology.

ISSN: 1618-3169

Imprint: Go"ttingen, Germany : Hogrefe & Huber, c2002-

Article: Gillard Ellen; Van Dooren Wim; Schaeken Walter; Verschaffel Lieven: Proportional reasoning as a heuristic-based process: time constraint and dual task considerations.

Volume: 56

Number: 2

Date: 2009

Pages: 92-99

Verified: <TN:92052><ODYSSEY:137.190.51.13/ILL> OCLC

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Proportional Reasoning as a Heuristic-Based Process

Time Constraint and Dual Task Considerations

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Abstract. The present study interprets the overuse of proportional solution methods from a dual process framework. Dual process theories claim that analytic operations involve time-consuming executive processing, whereas heuristic operations are fast and automatic. In two experiments to test whether proportional reasoning is heuristic-based, the participants solved "proportional" problems, for which proportional solution methods provide correct answers, and "nonproportional" problems known to elicit incorrect answers based on the assumption of proportionality. In Experiment 1, the available solution time was restricted. In Experiment 2, the executive resources were burdened with a secondary task. Both manipulations induced an increase in proportional answers and a decrease in correct answers to nonproportional problems. These results support the hypothesis that the choice for proportional methods is heuristic-based.

Keywords: proportional reasoning, word problem solving, dual process theory

Overuse of Proportionality

Proportional reasoning is a major tool for human beings to interpret everyday-life phenomena (Centre de Recherche sur l'Enseignement des Mathématiques, 2002; Lesh, Post, & Behr, 1988). Because of its usefulness for solving mathematics and science problems, it is also a key concept in education, where it is often practiced with missing-value word problems (in which three numbers, a, b, and c, are given, and a fourth has to be found by calculating the value of x in the equation $a/b = c/x$). In the long run, students' growing familiarity with, and competence in, proportional solution methods, together with the intrinsic simplicity and self-evident character of these methods, causes a deeply entrenched tendency to apply them also in situations where this is inadequate. This phenomenon has been demonstrated in different mathematical domains, such as elementary arithmetic, geometry, and probability (e.g., Cramer, Post, & Currier, 1993; De Bock, Van Dooren, Janssens, & Verschaffel, 2002; Fischbein, 1999; Fischbein & Schnarch, 1997; Van Dooren, De Bock, Depaepe, Janssens, & Verschaffel, 2003 – for an overview, see Van Dooren, De Bock, Janssens, & Verschaffel, 2008). For example, most sixth graders erroneously give the proportional answer "$30 \times 3 = 90$" instead of the additive answer "$30 + 10 = 40$" to the following nonproportional arithmetic problem (see Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005):

> Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run 5 laps, Kim has run 15 laps. When Ellen has run 30 laps, how many has Kim run?

Cramer et al. (1993) had showed that even 32 of 33 preservice elementary teachers made the proportional error.

In the present study, we tested the explanatory power of one of the most influential current frameworks in the psychology of reasoning and thinking (e.g., Evans, 2006), namely the dual process framework, for the overuse of proportionality with this type of missing-value arithmetic problems.

Dual Process Framework

One of the main themes of the cognitive reasoning research over the past decades is that human reasoning frequently violates traditional normative standards. A large body of research has demonstrated that in a wide range of reasoning tasks most educated adults fail to provide the response that is correct according to logic or probability theory (Evans, 2002; Kahneman, Slovic, & Tversky, 1982). Dual process theorists (e.g., Epstein, 1994; Evans & Over, 1996; Goel, 1995; Kahneman, 2000; Sloman, 1996; Stanovich, 1999) have tried to account for this gap between normative standards and actual performances by positing the existence of two distinct cognitive reasoning systems: A heuristic and an analytic system. The heuristic system is characterized as automatic, associative, unconscious, and undemanding of executive working memory capacity; the analytic system as consciously controlled, deliberate, and effortful. The heuristic system is assumed to operate in a fast and parallel way, and to generate answers based on similarity to stored prototypes (Sloman, 1996); the analytic system is assumed to be time-consuming and sequential, and to operate on "decontextualized" representations. Fast and unconscious heuristics often provide correct responses, but they can also bias

Experimental Psychology 2009; Vol. 56(2):92–99
DOI: 10.1027/1618-3169.56.2.92

reasoning in situations that require more elaborate, analytic processing. That is, the two systems will sometimes conflict and cue different responses. In these cases, a correct response can only be obtained when the analytic system overrides the responses generated by the heuristic system. Hence, a failure to provide the normatively correct answer may be attributed to the heuristic system's pervasiveness, and the analytic system's failure to intervene.

The dual process framework makes two important and strongly related processing claims. First, the heuristic system is assumed to operate faster than the analytic system (e.g., Epstein, 1994; Evans & Over, 1996; Goel, 1995; Kahneman, 2000; Sloman, 1996; Stanovich, 1999). In other words, with more time available, it is more likely that analytic processes will be engaged and that correct responses will be provided in cases wherein heuristic processes yield wrong answers. Second, the dual process framework assumes that heuristic and analytic reasoning differ in the involvement of executive working memory resources. The heuristic operations are posited to be automatic, hence undemanding of executive resources, whereas analytic operations heavily draw on such resources (e.g., Evans, 2002; Evans & Over, 1996; Stanovich & West, 2000). In other words, with more resources available, it is more likely that the analytic system will be engaged and that correct responses will be provided. Both processing claims have been successfully demonstrated for the most paradigmatic examples of experimental tasks that were developed and used within the dual process literature: The "Wason selection task" and the "Linda problem" (De Neys, 2006a).

Overuse of Proportionality Interpreted From a Dual Process Account

This study aimed to test the explanatory power of the dual process framework, and more specifically of the two above-mentioned processing claims, for the overuse of proportionality. So far, research within the dual process framework did not address tasks coming from ecologically more valid settings like mathematical word problem solving. Recently, however, Leron and Hazzan (2006) had argued that research on mathematical problem solving might benefit from taking a dual process stance. In this line, we claim that a dual process account may contribute to our understanding of students' overuse of proportionality in word problem solving. According to this account, due to the wide applicability of proportionality in daily life and due to the extensive instruction and training in proportional methods at school, a "proportional heuristic" is created in students, which is triggered by superficial features of word problems, such as the missing-value formulation. So, the proportional heuristic may provide students with correct responses to proportional missing-value word problems in a fast and almost effortless way. However, when solving nonproportional word problems with similar superficial features, the analytic system needs to override the proportional response generated by the heuristic system and needs to apply the right nonproportional solution method to the problem in order to provide the correct response. Note that we only assume that the heuristic processes determine the *selection* of the mathematical solution method; the actual computation of the numerical answer may, of course, still require some analytic processing.

When applying the two above-mentioned processing claims of the dual process framework to mathematical word problem solving, we expect that limiting students' resources, by restricting their response time or working memory capacity, will result in an increase in the choice of proportional solution methods. As these are correct for proportional problems, we anticipate no effect of restricting solution time or working memory capacity when solving them. For nonproportional problems, however, we expect an increase in the inadequate selection of proportional solution methods, and hence a decrease in performance. These predictions were tested in two experiments in which the participants had to solve two types of problems. First, problems for which proportional solution methods would provide a correct answer ("proportional problems") and, second, problems known to elicit incorrect answers based on proportionality ("nonproportional problems"). Experiment 1, addressed the first processing claim (i.e., the heuristic processes operate faster than the analytic processes), by experimentally manipulating the available solution time. Experiment 2 addressed the second processing claim (i.e., heuristic reasoning requires less executive working memory resources than analytic reasoning) by experimentally burdening executive resources with a concurrent attention-demanding secondary task.

Experiment 1

This experiment tested the heuristic character of proportional reasoning by focussing on the claim that the heuristic processes operate faster than the analytic processes. The participants had to solve a series of proportional problems and of nonproportional (additive) word problems known to elicit proportional answers. In the short condition, the available solution time was drastically reduced; in the control condition, no time limit was introduced. Assuming that the heuristic processes operate faster than the analytic processes (Evans & Over, 1996; Sloman, 1996; Stanovich & West, 2000), inadequate proportional answers to nonproportional problems ("proportional errors") would be provided faster than correct answers. So, if proportional errors need to be overridden by analytic processes to solve nonproportional problems correctly, we expect an increase in proportional errors when analytic processes have no time to successfully intervene. More specifically, we predict an increase in proportional answers for nonproportional problems under time constraint, resulting in a decrease in correct answers, whereas for proportional problems no significant effect is predicted.

Method

Participants

The participants were 167 students from the Faculty of Psychology and Educational Sciences from the University of Leuven, who participated in return for course credit.

Materials and Design

The word problem task consisted of 12 missing-value word problems in an open-answer format, presented in a quasi-randomized order. Six were proportional problems and six were nonproportional, additive word problems. The proportional and nonproportional problems were formulated as similarly as possible, and were controlled for length, number of syllables, technical reading complexity, and computational difficulty (number size and proportionality factor). Examples of proportional and nonproportional problems are given below:

Proportional:
Erik and Tom buy boxes of pencils in the shop. All boxes are equally expensive, but Erik buys fewer boxes.
Erik buys 4 boxes of pencils, while Tom buys 8 boxes. Knowing that Erik has to pay 24 Euros, how much does Tom have to pay?

Nonproportional:
Ellen and Kim are running around a track. They run equally fast but Ellen started later.
When Ellen has run 5 laps, Kim has run 15 laps.
When Ellen has run 30 laps, how many has Kim run?

The available solution time was experimentally manipulated as a between participant variable. We wanted the participants in the short condition to have sufficient time to respond but not enough for analytic processes to intervene, and the participants in the control condition to have sufficient time to respond thoughtfully without feeling any time pressure. Therefore, in the control condition, there was no time limit, whereas the participants in the short condition had only 17 s (i.e., the median time of proportional errors in a pilot study without time constraint).

Procedure

The participants were randomly assigned to the control (n = 100) or the short condition (n = 67), and they worked individually on a computer in a computer room. They first received general instructions about the task. In the short condition, the participants also received instructions about the time limit. The bottom of their screen showed a bar indicating elapsed and remaining time. Time started running as soon as the word problem appeared. In both conditions, the procedure was clarified by one practice item (which was unrelated to the experimental items). Finally, the participants received 12 experimental problems. They did not have to type in their calculations, but were asked to merely type their final solution. Between problems, a rest screen was presented. Pressing the enter key automatically led to the next problem. In the short condition, when time had elapsed without an answer, the problem disappeared and the rest screen was presented.

Results

In the short condition, trials in which no response was given within time were removed from the analysis. This resulted in an elimination of almost 6% (46 of 804 trials) of the data in this condition. Additionally, when no response at all was given, trials were also removed from the analysis. This resulted in an additional elimination of less than 1% of the data (5 of 1,200 trials in the control and 3 of 804 trials in the short condition). Answers were classified as either *correct* (i.e., the correct response was given), *proportional error* (i.e., a proportional response to a nonproportional item) or *other error* (i.e., another response). Because the participants were not asked to give their calculations, pure miscalculations could not be identified. They were classified as other errors rather than as correct answers or proportional errors.

Table 1. Mean proportion of correct answers for Type of word problem and Time condition (Experiment 1)

	Type of word problem			
	Proportional		Nonproportional	
Time	Mean	n	Mean	n
Long	0.97	598	0.62	597
Short	0.86	368	0.43	387
Total	0.93	966	0.55	984

We ran a logistic regression analysis with word problem type (proportional vs. nonproportional) and time condition (control vs. short) as factors and accuracy as dependent variable. First, we observed a main effect for word problem type: Accuracy for proportional problems (93%) was significantly higher than for nonproportional problems (55%; see Table 1), $\chi^2(1) = 254.62$, $p < .01$. There was also a main effect of time condition: Time constraint led to a general decrease in accuracy, $\chi^2(1) = 60.87$, $p < .01$. Further, an interaction effect between word problem type and time condition was observed, $\chi^2(1) = 9.95$, $p < .01$. Under time constraint, accuracy decreased stronger for nonproportional (from 62% to 43%), $\chi^2(1) = 33.41$, $p < .01$, than for proportional problems (from 97% to 86%), $\chi^2(1) = 35.80$, $p < .01$.

Errors in nonproportional problems were mostly proportional. Moreover, under time constraint, proportional errors significantly increased from 34% to 43%, $\chi^2(1) = 8.49$, $p < .01$. Time constraint also led to small increases in other errors, but comparably for proportional (from 3% to 14%) and nonproportional problems (from 3% to 13%).

An analysis at item level revealed that the obtained effects were not caused by a specific subset of items, but were spread evenly across all items. Indeed, for all items accuracy decreased under time constraint, however this decrease was smaller for each proportional problem (11% on average) than for each nonproportional problem (19% on average). Also, proportional errors increased under time constraint for each nonproportional problem (9% on average).

An analysis at the participant level revealed that although not all the participants overused proportionality, more participants in the short condition did so (see Figure 1). The participants were categorized into three groups according to their performance on nonproportional problems: Proportional reasoners (who never answered correctly, but instead made proportional errors and, occasionally, another error), correct reasoners (who never made a proportional error but instead

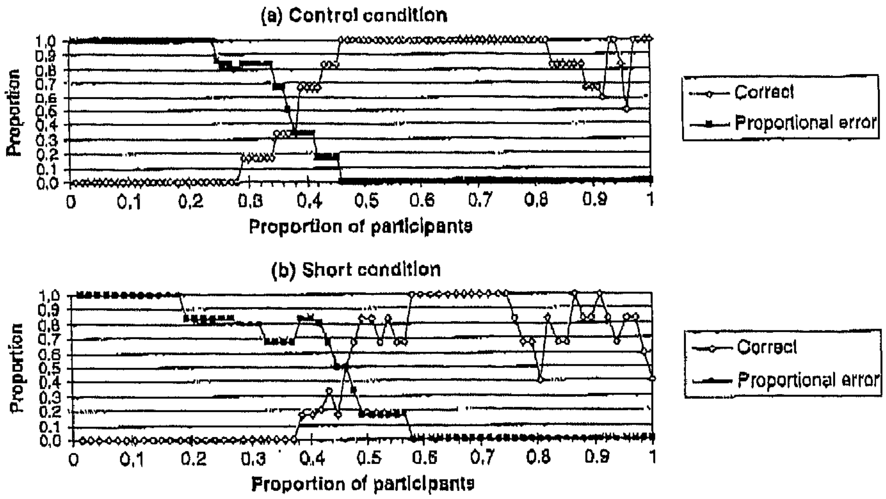

Figure 1. Individual profiles in the (a) control condition and (b) short condition.

reasoned correctly and occasionally made another error) and a remaining group (with minimally one proportional error and one correct nonproportional answer). This categorization revealed that the correct reasoners' group was larger in the control condition (53%) than in the short condition (42%), indicating that more people made minimally one proportional error. Also, the proportional reasoners' group was larger in the short (37%) than in the control condition (28%).

Discussion

The results, first, confirm that even adults strongly tend to overuse proportional methods. More importantly, they support our main hypothesis that people make more errors under time constraint, particularly more proportional errors. Hence, our results indicate that correct nonproportional reasoning is more time-consuming than (incorrect) proportional reasoning.

Clearly, our experimental manipulation could only affect the participants who would not already perform at floor level without time constraint. The analysis at the participant level revealed, however, that there were quite a number of proportional reasoners in the control condition. Moreover, in the short condition there was also a subset of the participants who always kept reasoning correctly under time constraint and, so, were unaffected by the experimental manipulation. But, most importantly, we found that more people tended to make proportional errors in the short condition than in the control condition.

Experiment 2

In the second experiment, we investigated whether the second processing claim concerning the role of working memory could also be demonstrated for proportional reasoning. This time, we experimentally burdened the participants' executive resources with an attention-demanding secondary task that needed to be accomplished simultaneously. More specifically, we combined the word problem task with the dot memory task (Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001). Again, the participants had to solve proportional and nonproportional (additive) problems. An experimentally burdened working memory could be expected to elicit more proportional errors, without affecting the answers to proportional problems.

Method

Participants

The participants were 148 students from the Faculty of Psychology and Educational Sciences from the University of Leuven. They participated in return for course credit. None of them had participated in the previous study.

Materials and Design

Word Problem Task

The word problem task was the same as in Experiment 1.

Dot Memory Task

The dot memory task is a classic spatial storage task (e.g., Bethell-Fox & Shepard, 1988; Miyake et al., 2001). A three by three matrix containing three to four dots was briefly presented, which the participants had to memorize and reproduce afterwards.

In the load condition, the matrix was filled with complex four-dot patterns (a "two-piece" or "three-piece" pattern based on Bethell-Fox & Shepard, 1988; Verschueren, Schaeken, & d'Ydewalle, 2004; e.g., see Figure 1). Miyake et al. (2001) had showed that this memorization task burdens executive resources. In the control group, the patterns consisted of three dots in one line ("one-piece" patterns, based

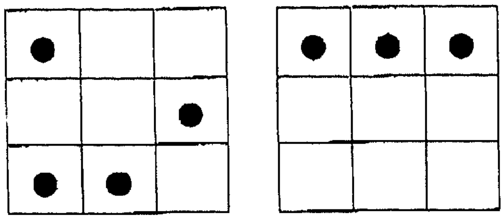

Figure 2. Examples of the dot patterns in the load condition (left) and the control condition (right).

on the work of De Neys, 2006b; e.g., see Figure 2), which has proven to burden executive resources only minimally.

Procedure

The participants were randomly assigned to the control (n = 63) or to the load condition (n = 85). They worked individually on a computer in a computer room. The experiment first introduced the dot memory task. After a practice item, the participants received instructions for the word problem task. It was explicitly mentioned that a correct dot pattern reproduction was crucial. We did this because we wanted the participants to actively attend to the dot pattern and to try to memorize it as well as possible.

First, the dot pattern was presented for 1 s. Next, the word problem appeared. The participants typed their answer to the word problem and pressed the enter key. Thereafter, the empty matrix was presented and the participants had to reproduce the dot pattern using the numeric keypad, with each key corresponding to a square in the matrix (1 = lower left and 9 = upper right). When a key was pressed, a dot appeared in the corresponding location. Pressing the key once more removed the dot. Feedback was only given on dot pattern reproduction. Before each trial, a rest screen was presented. Pressing the enter key started the next trial. As in Experiment 1, the entire procedure was clarified with one practice item. The 12 problems (6 proportional and 6 nonproportional) appeared in a quasi-random order.

Results

Dot Memory Task

The mean number of correctly localized dots was 3.58 (SD = 0.77) for the complex four-dot pattern in the load condition and 2.85 (SD = 0.60) for the simple three-dot pattern in the control condition. The mean accuracy rate was 72% (SD = 0.45) in the load condition and 94% (SD = 0.24) in the control condition. The participants whose mean number of correctly localized dots was more than three standard deviations below the overall mean were eliminated from the sample. This resulted in an elimination of 34 participants from the load condition and 19 participants from the control condition, leaving respectively 51 and 44 participants in both conditions. This elimination was necessary because those participants' lower performance on the dot memory task may reflect a lower executive burden for them than for the other participants in the same condition.

Word Problem Task

Unanswered trials were removed from the analysis, resulting in an elimination of < 1% of the data (2 of 612 trials in the load and 1 of 528 trials in the control condition). We ran a logistic regression analysis with word problem type (proportional vs. nonproportional problems) and working memory load (load vs. control condition) as factors and accuracy as dependent variable. First, we observed a main effect of word problem type: Accuracy for proportional problems was significantly higher (97%) than for nonproportional problems (68%; see Table 2), $\chi^2(1) = 94.69$, $p < .01$.

Second, the accuracy rates in Table 2 showed that working memory load affects correct nonproportional reasoning but has no effect on correct proportional reasoning. Under working memory load, accuracy for nonproportional word problems decreased from 76% to 62%, $\chi^2(1) = 12.09$, $p < .01$, while for proportional word problems no decrease was observed. (Actually, performance even slightly increased from 96% to 98%, $\chi^2(1) = 2.41$, $p = .12$.) This was confirmed by a significant interaction effect between working memory load and word problem type, $\chi^2(1) = 6.64$, $p < .01$, and by the absence of a main effect of working memory load $\chi^2(1) = 0.13$, $p = .71$.

The errors in nonproportional word problems were mostly proportional errors. Proportional errors significantly increased under load from 22% to 34%, $\chi^2(1) = 9.78$, $p < .01$. Only few other errors were made in proportional as well as nonproportional problems, with no difference between the load and control condition.

An analysis at item level revealed that the obtained effects were not caused by a specific subset of items, but were spread evenly across all items. Indeed, accuracy decreased under load for each nonproportional item separately (13% decrease on average), and proportional errors increased accordingly (average increase of 11%). For some proportional problems accuracy decreased under load (4% decrease on average), whereas for others accuracy increased (4% increase on average).

An analysis at the participant level revealed that although not all the participants overused proportionality, more did so in the short condition (see Figure 3). We again categorized the participants into three groups, using the same criteria as in Experiment 1. This categorization revealed that the correct

Table 2. Mean proportion of correct answers for Type of word problem and Load condition (Experiment 2)

	Type of word problem			
	Proportional		Nonproportional	
Load	Mean	n	Mean	n
Low	0.96	263	0.76	264
High	0.98	304	0.62	306
Total	0.97	567	0.68	570

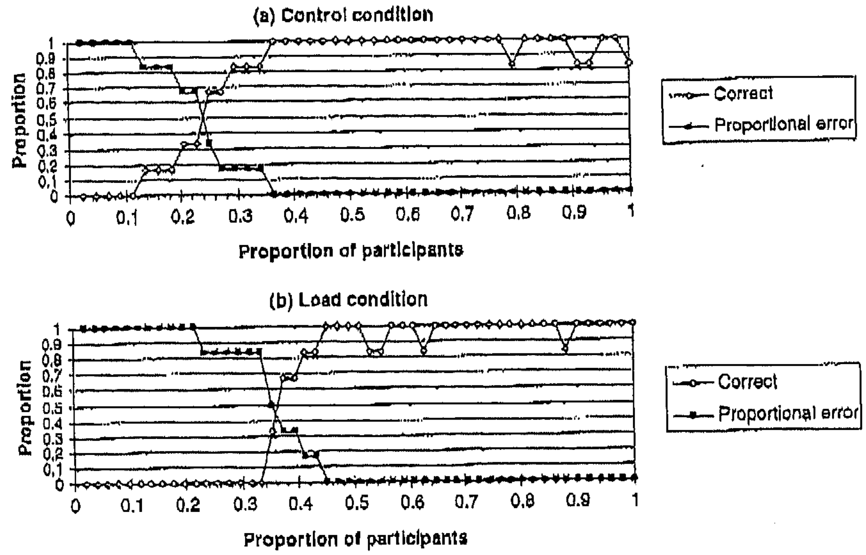

Figure 3. Individual profiles in the (a) control condition and (b) load condition.

reasoners' group was larger in the control (63%) than in the load condition (55%), indicating that under working memory load, more people tend to make at least one proportional error. Indeed, in the load condition, the proportional reasoners' group (33%) and the remaining group (12%) together are larger than in the control condition (respectively 26% and 11%).

Discussion

First of all, as in Experiment 1, the results indicate a strong tendency in adults to overuse proportional methods. More importantly, the results supported our main hypothesis that under working memory load, more proportional errors are made, whereas performance on proportional problems remains unaffected. So, the results indicate that processes underlying correct nonproportional reasoning are more demanding of working memory capacity than those underlying (incorrect) proportional reasoning.

Clearly, as in Experiment 1, our manipulation could only affect those participants who would not already perform at floor level without working memory load, while the analysis at the participant level revealed that such a group of proportional reasoners does exist. A subset of the participants also remained unaffected by the experimental manipulation, namely the correct reasoners' group in the load condition. But, most importantly, in the load condition, more people made proportional errors in the load condition than in the control condition.

General Discussion

This study aimed to test the explanatory power of the dual process framework for the overuse of proportionality. This was done by focussing on two important processing claims of the dual process framework: The first claim concerns the differential processing speed of the two processes and the second addresses the differential involvement of executive resources in the two processes.

Experiment 1 yielded evidence for our assumption that, due to the extra time needed for the analytic system, correct answers to nonproportional word problems take more time than (inadequate) proportional answers. Although under time constraint, accuracy also decreased for proportional problems, the decrease was larger for nonproportional problems. Also as expected, this stronger decrease was caused by the incorrect application of proportional solution methods. Experiment 2 showed that correct responses to nonproportional problems require more executive resources than (incorrect) proportional responses: Under working memory load, accuracy for nonproportional problems decreased, whereas performance for proportional problems remained unaffected. So, in conclusion, incorrect proportional reasoning on nonproportional problems as well as correct proportional reasoning on proportional problems relies on fast and undemanding heuristic processes, whereas correct nonproportional reasoning on nonproportional problems is based on slow and demanding analytic processes. Hence, we can conclude that the dual process theory can be successfully used to understand the mechanism behind the overuse of proportionality. However, some issues remain unresolved.

First, the methods that are used also have disadvantages. In both experiments, an analysis at the participant level revealed that a large group of the participants already consistently overused proportional methods in the control condition. Of course, the participants who already consistently make proportional errors without any heuristic-stimulating manipulation (i.e., time constraint or working memory load) cannot be affected by these manipulations. So, such participants cannot contribute to the observed experimental effect. Nevertheless, in both experiments, the number of consistent proportional reasoners increased considerably under the experimental manipulation. Moreover, there was a group of the participants who solved all nonproportional problems correctly, but in each experiment, this group was smaller in the experimental condition than in the control condition. This (the fact that a group of people consistently overuse proportionality without any heuristic-stimulating manipulation and that another group is still able to reason consistently correct under those manipulations) does not need to imply that the dual process framework cannot provide an explanation for all the participants. Thinking in terms of individual differences in working memory capacity, we hypothesize that the consistently correct reasoners were the people on the higher end of the working memory capacity continuum, and that the strength of the manipulations (i.e., the amount of available solution time in Experiment 1 and of working memory load in Experiment 2) was not sufficient to hinder their correct reasoning. The consistent proportional reasoners in the control conditions, on the other hand, probably were on the lower end of the continuum. Comparison of the first and the second experiment indicates that this might indeed be the case. In Experiment 2, we eliminated the participants performing very weakly on the secondary task, because the manipulation most probably did not work for them individually. In doing so, however, we may have eliminated the participants on the lower end of the working memory capacity continuum, maintaining proportionally more participants from the higher end than in the first experiment. Indeed, in Experiment 2, the correct reasoners' group was larger than in Experiment 1, whereas the proportional reasoners' group was smaller. In future research, we will test this assumption by measuring the participants' working memory capacity as well. Further, it might also be interesting to personalize the strength of the experimental manipulations.

Second, certain authors have indicated that within the dual process literature there is a need for a better understanding of how the processes operate precisely (De Neys & Glumicic, 2008; Evans, 2007; Osman, 2004; Stanovich & West, 2000). An important issue is the characterization of the conflict monitoring process. Different views exist on the efficiency of the conflict monitoring component of the analytic system, resulting in different characterizations of the nature of the reasoning error. The basic question is: Do people fail to *detect* a conflict (Kahneman, 2003) or are they only unable to *inhibit* the dominant heuristics (Epstein, 1994; Sloman, 1996)? These distinct views on conflict monitoring also result in different characterizations of the interaction between the analytic and heuristic systems (Evans, 2007): According to some authors (e.g., Kahneman, 2003), the two systems operate in a serial mode whereas others (Epstein, 1994; Sloman, 1996) claim that they operate in parallel. Recently, interesting work has been done within the domain of belief bias, using classic base rate neglect problems (De Neys and Glumicic, 2008), indicating that for this task the problem rather lies with failing response inhibition than with failing conflict detection, and consequently, that there must be at least a stage where both processes operate in parallel (conflict detection requires analytic reasoning). However, this conclusion needs further investigation, especially with respect to its generalizability to other tasks. Possibly, for the overuse of proportionality in word problem solving, the problem is one of the failure of conflict detections instead of response inhibitions.

Further, it has been argued that the proposed dichotomy between heuristic and analytic processes does not adequately cover the range of processes that are identified in reasoning research (Osman, 2004; Osman & Stavy, 2006). For example, according to Osman and Stavy, skill-based reasoning (e.g., proportional reasoning) shares features typical of both heuristic and analytic processes. Skill-based reasoning is first acquired explicitly (analytic processing), and through practice it is applied and generalized automatically (heuristic processing) to a variety of task domains. Also, the all-or-none distinction between automatic and nonautomatic processes has been criticized (see Moors & De Houwer, 2006). Studies have demonstrated processes (such as Stroop interference) that have features typical of both nonautomatic and automatic processes. In light of these criticisms, further specification is needed of which features of heuristic processes – and even more detailed, which features of automatic processes – exactly apply to proportional reasoning (and which do not).

Given that proportional reasoning can be considered as skill-based, it might also be interesting to investigate whether the same pattern of results can be found in children. In a first pilot study (Gillard, Van Dooren, Schaeken, & Verschaffel, 2007), we presented a group of 11–12-year-old children with the same set of proportional and nonproportional (additive) word problems. As it turned out quite hard for these pupils to calculate solutions by means of mental arithmetic (without paper and pencil), we used a forced choice task, instead of an open-answer task as in the studies reported above. More specifically, we presented all word problems together with two possible solutions (a proportional and a nonproportional, additive one), which they had to accept or reject. The available time for solving the problems was manipulated similar to that in Experiment 1. But since we observed problematic floor effects for the nonproportional problems in the control condition, it was highly unlikely that we would find a decrease in correct answers in the short condition. In fact, floor effects among this age group were not surprising, since a previous study (Van Dooren et al., 2005) had shown that 11–12-year-old children are most prone to the overuse of proportional solution methods. So, manipulations that stimulate heuristic processes can be expected to have little effect on this age group. Therefore, to verify the heuristic character of proportional reasoning in children, it might be necessary to search for experimental manipulations that stimulate analytic processes instead of manipulations that inhibit them.

In sum, the results of the present study indicate that the dual process theory provides a fruitful framework for understanding and explaining the overuse of proportionality and, more generally, indicate that it is a promising framework for investigating other reasoning errors in mathematics and science as well, as suggested by Leron & Hazzan (2006). However, further refinement and validation of several claims related to the theory, such as specifying the interaction between the analytic and heuristic processes, determining the nature of the reasoning error, as well as further testing the applicability of the framework to children, is necessary, before this framework can truly establish its usefulness.

Acknowledgement

This research was partially supported by Grant GOA 2006/01 "Developing adaptive expertise in mathematics education" from the Research Fund of the K. U. Leuven, Belgium. Wim Van Dooren is a post-doctral researcher of the Research Fund K. U. Leuven.

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Received May 2, 2007
Revision received November 16, 2007
Accepted December 28, 2007

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