Math 1050 - Exam 4a Name________________________________

Fall 09

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

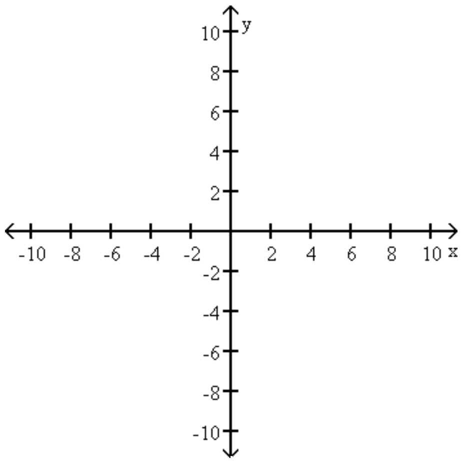
Solve the problem. (4 pts)

1) A hall 130 feet in length was designed as a whispering gallery (it is in the shape of an ellipse.) If the ceiling is 25 feet high at the center, how far from the center are the foci located? 1) ________

A) 60 ft B)

Find the center, axis, vertices, and foci of the equation, then sketch the graph. (8 pts)

2) $x^2 - 16y^2 + 6x + 128y - 263 = 0$ 2) ________



A)

B) center at (-3, 4)
transverse axis is parallel to x-axis
vertices at (-7, 4) and (1, 4)
foci at $(-3 - \sqrt{17}, 4)$ and $(-3 + \sqrt{17}, 4)$
asymptotes of $y - 4 = -\frac{1}{4}(x + 3)$ and $y - 4 = \frac{1}{4}(x + 3)$

C)

D)

Solve the system of equations using any matrix method. (8 pts)

3) 3) ________

$$\begin{cases} x + y + z = 10 \\ x - y + 4z = 23 \\ 2x + y + z = 14 \end{cases}$$

A) B) x = 4, y = 1, z = 5; (4, 1, 5)

C) x = D)

Solve for x. (6 pts)

4) 4) ________

$$\begin{vmatrix} 5 & -3 & 1 \\ -2 & -2 & x \\ 8 & 2 & -1 \end{vmatrix} = 28$$

A) 0

Show that the matrix has no inverse. (8 pts)

5) $\begin{bmatrix} 2 & 10 & 4 \\ -3 & -1 & 1 \\ -1 & 7 & 4 \end{bmatrix}$ 5) ________

A) $\left[\begin{array}{ccc|ccc} 2 & 10 & 4 & 1 & 0 & 0 \\ -3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 7 & 4 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & \frac{1}{2} & 0 & 0 \\ -3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 7 & 4 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 14 & 7 & \frac{3}{2} & 1 & 0 \\ -1 & 7 & 4 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 14 & 7 & \frac{3}{2} & 1 & 0 \\ 0 & 12 & 6 & \frac{1}{2} & 0 & 1 \end{array}\right] \rightarrow$

$\left[\begin{array}{ccc|ccc} 1 & 5 & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{3}{28} & \frac{1}{14} & 0 \\ 0 & 12 & 6 & \frac{1}{2} & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{3}{28} & \frac{1}{14} & 0 \\ 0 & 0 & 0 & -\frac{11}{14} & -\frac{6}{7} & 1 \end{array}\right]$

B)

Solve the system using any matrix method. (5 pts)

6) $\begin{cases} x + 2y + 3z = 7 \\ x + y + z = 10 \\ 2x + 2y + z = 2 \end{cases}$ 6) ________

The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 4 & -1 \\ 1 & -5 & 2 \\ 0 & 2 & -1 \end{bmatrix}$.

A)
B)
C)
D) $x = 31$, $y = -39$, $z = 18$; $(31, -39, 18)$

Write out the first five terms of the sequence. (4 pts)

7) $\{s_n\} = \{n^2 - n\}$ 7) ________

A) $s_1 = 0$, $s_2 = 2$, $s_3 = 6$, $s_4 = 12$, $s_5 = 20$
B)
C)
D)

Find the indicated term of the arithmetic sequence. (5 pts)

8) The 10th term of an arithmetic sequence with third term 2 and 6th term -2.5. 8) ________

A) -8.5 B) C) D)

Find the sum. (5 pts each)

9) $\sum_{n=1}^{25} (3n - 7)$ 9) ________

A) B) 800 C) D)

FInd the infinite sum if it exists. (6 pts)

10) $1 - \frac{1}{3} + \frac{1}{9} - \cdots$ 10) _______

A) Converges; $\frac{3}{4}$ B) C) D)

Solve. (8 pts)

11) Kurt deposits $150 each month into an account paying annual interest of 6% compounded monthly. How long will it take to accumulate $27,948 in his account? 11) _______

A) B) C) D) 11 years

Use the Principle of Mathematical Induction to show that the statement is true for all natural numbers n. (8 pts)

12) $4 + 9 + 14 + ... + (5n - 1) = \frac{n}{2}(5n + 3)$ 12) _______

A) First we show that the statement is true when n = 1.

For n = 1, we get $4 = \frac{(1)}{2}(5(1) + 3) = 4$.

This is a true statement and Condition I is satisfied.

Next, we assume the statement holds for some k. That is,

$4 + 9 + 14 + ... + (5k - 1) = \frac{k}{2}(5k + 3)$ is true for some positive integer k.

We need to show that the statement holds for k + 1. That is, we need to show that

$4 + 9 + 14 + ... + (5(k + 1) - 1) = \frac{k + 1}{2}(5(k + 1) + 3)$.

So we assume that $4 + 9 + 14 + ... + (5k - 1) = \frac{k}{2}(5k + 3)$ is true and add the next term,

$5(k + 1) - 1$, to both sides of the equation.

$$\begin{aligned}
4 + 9 + 14 + ... + (5k - 1) + 5(k + 1) - 1 &= \frac{k}{2}(5k + 3) + 5(k + 1) - 1 \\
&= \frac{1}{2}[k(5k + 3) + 10(k + 1) - 2] \\
&= \frac{1}{2}(5k^2 + 3k + 10k + 10 - 2) \\
&= \frac{1}{2}(5k^2 + 13k + 8) \\
&= \frac{1}{2}(k + 1)(5k + 8) \\
&= \frac{k + 1}{2}(5k + 5 + 3) \\
&= \frac{k + 1}{2}(5(k + 1) + 3)
\end{aligned}$$

Conditions II is satisfied. As a result, the statement is true for all natural numbers n.

B)