Math 4710 HOMEWORK #9 Name:

Due 4/21/2023, 10:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 10.4 #10
   Hint: See the class notes.
2. Sec 10.6 #18
   Hint: Apply the property $\mathcal{F}(f(x)) = \frac{1}{\pi}\frac{\sin(a\omega)}{\omega}$ for $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$, with $a = ct$. The final answer should be the same as the d'Alembert's solution.

   Hint: Use Fourier cosine transform.
3. Sec 10.5 #11(a)
4. Sec 10.6 #4(a)
   Hint: Rather than finding the inverse Fourier sine transform of $U_S(\omega, y)$ directly, find the inverse Fourier sine transform of $\frac{\partial U_S}{\partial y}$: $\frac{\partial u}{\partial y}(x, y) = \mathcal{S}^{-1}(\frac{\partial U_S}{\partial y}) = \mathcal{S}^{-1}(F_S(\omega)\, e^{-\omega y})$. Then $u(x, y) = \int \frac{\partial u}{\partial y}(x, y)\, dy + c$. The final answer is $u(x\,, y) = \frac{1}{2\pi}\int_0^{\infty} f(\xi) \ln \frac{(x-\xi)^2+y^2}{(x+\xi)^2+y^2}\, d\xi$.
5. Sec 9.5 # 2
   Note: x_0 and y_0 are switched in part (b). Hint: See class notes.
6. Sec 9.5 # 3(a)
   Hint: Follow subsection 9.5.3 or class notes. The eigenpairs are $\lambda_{nm} = [\frac{(2n-1)\pi}{2L}]^2 + (\frac{m\pi}{H})^2$, $\phi_{nm}(x, y) = \sin\frac{(2n-1)\pi x}{2L} \cos\frac{m\pi y}{H}$ for $n = 1, 2, \cdots$ and $m = 0, 1, \cdots$.
7. Sec 9.5 # 4
   Hint: Even though the boundary condition is not homogeneous, still use the Green's function with the properties $\nabla^2 G = \delta(\mathbf{x} - \mathbf{x}_s)$ in the given region and $G = 0$ on the boundary of the given region. Apply the three-dimensional version of Green's 2nd identity.
8. Sec 9.5 # 6
   Hint: For part (a), apply the Green's 2nd identity for functions u and ϕ_h. For part (b), assume $u = \sum_{\lambda} a_\lambda\, \phi_\lambda$ where $\nabla^2\phi = -\lambda\, \phi$. Show a_0 can be chosen arbitrarily where $\iint_\Omega f\, \phi_h\, dA = 0$ or $\lambda = 0$
9. Free points!
10. Free points!