Math 4710 HOMEWORK #8 Name:

Due 4/7/2023, 10:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 10.3 #8
2. Sec 10.4 #3(a)
   Hint: Use the properties $\mathcal{F}(f(x-\beta)) = e^{i\omega\beta}F(\omega)$ and $\mathcal{F}\left(\sqrt{\frac{\pi}{\beta}}\, e^{-\frac{x^2}{4\beta}}\right) = e^{-\beta\omega^2}$, before applying the Convolution Theorem.
3. Sec 10.4 #4(a)
   Hint: Use the linearity property of the inverse Fourier transform, $\mathcal{F}^{-1}(e^{-\gamma t}H(\omega)) = e^{-\gamma t}\mathcal{F}^{-1}(H(\omega))$, before applying the Convolution Theorem.
4. Show that $\mathcal{F}(f'(x)) = -i\,\omega\,\mathcal{F}(f(x))$, assuming $|f(x)| \to 0$ as $|x| \to \infty$.
   Hint: See the class notes or section 10.4.
5. Show that $\mathcal{F}^{-1}(F(\omega)G(\omega)) = \frac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} f(\xi)g(x-\xi)\,d\xi$, where $F(\omega) = \mathcal{F}(f(x))$ and $G(\omega) = \mathcal{F}(g(x))$.
   Hint: See the class notes or section 10.4.
6. Sec 10.4 #8
   Hints: See the class notes. Since the hyperbolic sine is an odd function and $|\omega| = \pm\omega$, we can write $U(x,\omega) = A(\omega)\sinh(|w|x) + B(\omega)\sinh(|w|(L-x))$ as $U(x,\omega) = A(\omega)\sinh(wx) + B(\omega)\sinh(w(L-x))$ by incorporating the minus signs into A and B, as needed. Also, $\mathcal{F}^{-1}\left(\frac{\sinh(\alpha\omega)}{\sinh(\beta\omega)}\right) = \frac{\pi}{\beta}\frac{\sin(\frac{\alpha\pi}{\beta})}{\cosh(\frac{\pi y}{\beta})+\cos(\frac{\alpha\pi}{\beta})}$, if $0 < \alpha < \beta$, and Fourier transform was with respect to the y variable.
7. Sec 10.4 #9
   Hints: Assume $|u| \to 0$ as $y \to \infty$. See the class notes.
8. Use the Fourier transform to solve the ODE $y''(x) - b^2\,y(x) = f(x)$, $-\infty < x < \infty$.
   Hint: $\mathcal{F}(e^{-\beta|x|}) = \frac{\beta}{\pi(\omega^2+\beta^2)}$.
9. Free points!
10. Free points!