Math 4710 HOMEWORK #5 Name:

Due 3/3/2023, 10:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet.

1. Solve

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\,\frac{\partial^2 u}{\partial x^2} - \frac{x}{L}, && 0 < x < L,\ t > 0 \\ u(0,\,t) &= 0, && t > 0 \\ u(L,\,t) &= 0, && t > 0 \\ u(x,\,0) &= 0, && 0 < x < L \end{aligned}$$

Hint: Use $u_E = \frac{1}{6kL}\,x^3 - \frac{L}{6k}\,x$. Find the constants. You may use Mathematica and results of an earlier problem!

2. Solve

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2\,\frac{\partial^2 u}{\partial x^2}, && 0 < x < L,\ t > 0 \\ u(0,\,t) &= 0, && t > 0 \\ u(L,\,t) &= 0, && t > 0 \\ u(x,\,0) &= f(x), && 0 < x < L \\ \frac{\partial u}{\partial t}(x,\,0) &= g(x), && 0 < x < L \end{aligned}$$

using the method of seperation of variables.
Hint: You may use the Review, Identities, Theorems, Formulas and Tables hand-out.

3. Sec 8.2 #6(a)
Hint: $u_E'' = 0$. You may use results of an earlier problem!

4. Solve

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2\,\frac{\partial^2 u}{\partial x^2}, && 0 < x < L,\ t > 0 \\ u(0,\,t) &= 0, && t > 0 \\ u(L,\,t) &= t, && t > 0 \\ u(x,\,0) &= \tfrac{1}{6}\,x^3 - \tfrac{L^2}{6}\,x, && 0 < x < L \\ \frac{\partial u}{\partial t}(x,\,0) &= \tfrac{x}{L}, && 0 < x < L \end{aligned}$$

Hint: $r(x,\,t) = \frac{x}{L}\,t$. Find the constants. You may use Mathematica and results of an earlier problem!

5. Sec 8.3 #1(c)
Hints: Choose $r(x,\,t) = A(t)$. The related eigenfunctions are $\phi_n(x) = \sin\frac{(2n-1)\pi x}{2L}$.

6. Sec 8.3 #7
Hints: Choose $r(x,\,t) = \frac{x}{L}\,t$. Apply the method of eigenfunction expansion to the remaining problem with the homogeneous boundary condition. Don't restate the solution of problem 2!

7. Sec 8.3 #6
Hints: For $n \neq 5$, $a_n'(t) = -n^2 a_n(t)$ and for $n = 5$, $a_5'(t) = -25a_5(t) + e^{-2t}$.

8. Prove $\int_a^b [uL(v) - vL(u)]\, dx = p(x)\left(u\frac{dv}{dx} - v\frac{du}{dx}\right)\Big|_a^b$ where $L(u) = \frac{d}{dx}\left[p(x)\frac{du}{dx}\right] + q(x)\,u$.
   Hint: See the lecture notes.

9. Sec 5.5 #1(b)
   Note: $L(u) = \frac{d}{dx}[p(x)\frac{du}{dx}] + q(x)\,u$.

10. Free points!