Math 4710 HOMEWORK #2 Name:

Due 2/3/2023, 10:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 12.6 #3 The characteristic through $x = 0$, $t = t_0$ is $x = c_0(t - t_0)$ on which $\rho(x, t) = g(t_0)$.
2. Sec 12.6 #7(a) Hint: In the fan-like region, $\rho = +\sqrt{\frac{x}{t}}$ since for $x = 9t$, $\rho = 3$.
3. Sec 12.6 #7(b)
4. Sec 12.6 #19(a) Hint: The shock occurs at $x = 0$ and $t = 0$ and the shock wave satisfies $\frac{dx}{dt} = \frac{[\frac{1}{3}\rho^3]}{[\rho]} = \cdots = \frac{37}{3}$, $x(0) = 0$.
5. Sec 12.6 #19(b) Hint: The shock occurs at $x = 1$ and $t = 0$ and the shock wave satisfies $\frac{dx}{dt} = \frac{[2\rho^2]}{[\rho]} = \cdots = 10$, $x(0) = 1$.
6. Sec 12.6 #6(a) Hint: $c(\rho) = q'(\rho) = \frac{d}{d\rho}[\rho\, u(\rho)] = \cdots = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$. See the class notes.
7. Sec 12.6 #6(b) Hint: $c(\rho) = q'(\rho) = \frac{d}{d\rho}[\rho\, u(\rho)] = \cdots = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$. There are two fan-like regions. For both $\frac{dx}{dt} = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$ where $\rho$ is a constant value on the characteristic. The initial conditions for the two regions are $x(0) = 0$ and $x(0) = a$.
8. Sec 12.6 #6(c) Hint: $c(\rho) = q'(\rho) = \frac{d}{d\rho}[\rho\, u(\rho)] = \cdots = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$. The fan-like region is $-\frac{1}{5}u_{\max}t < x < \frac{3}{5}u_{\max}t$.
9. Sec 12.6 #17(a) Hint: $c(\rho) = q'(\rho) = \frac{d}{d\rho}[\rho\, u(\rho)] = \cdots = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$. The shock occurs at $x = 0$ and $t = 0$ and the shock wave satisfies $\frac{dx}{dt} = \frac{[q(\rho)]}{[\rho]} = \cdots = \frac{1}{5}u_{\max}$, $x(0) = 0$, where $q(\rho) = u_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right)$.
10. Sec 12.6 #17(b) Hint: $c(\rho) = q'(\rho) = \frac{d}{d\rho}[\rho\, u(\rho)] = \cdots = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)$. The shock occurs at $x = 0$ and $t = 0$ and the shock wave satisfies $\frac{dx}{dt} = \frac{[q(\rho)]}{[\rho]} = \cdots = 0$, $x(0) = 0$, where $q(\rho) = u_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right)$.