

Due 10/27/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. See 2.6 #6
2. See 2.6 #10
3. Sec 2.7 #1
4. Sec 2.7 #3(d) Notes: To match our notes, use λ in place of λ^2 . You may assume λ is a real number. Apply the S-L theorem. Find the approximate value of the first few eigenvalues by setting $a = 1$ and using Mathematica. Then use them to graph the first few eigenfunctions, using Mathematica.

Consider the following eigenvalue problem for the next two problems.

$$\begin{cases} \phi''(x) - q(x)\phi(x) = -\lambda\phi(x), & 0 < x < a \\ \phi'(0) = 0 \\ \phi(a) = 0 \end{cases}$$

where $q(x)$ is a continuous nonnegative function on $0 \leq x \leq a$.

5. Prove that the eigenfunctions of this EVP are orthogonal. Note: Don't just quote a S-L theorem. Derive the orthogonality property as we did in class.
6. Prove that the eigenvalues of this EVP are nonnegative. Note: Don't just quote a S-L theorem. First derive the Raleigh Quotient as we did in class.
7. Free points!
8. Free points!
9. Free points!
10. Free points!