Math 3710   HOMEWORK #8   Name:

Due 10/20/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. See 2.5 #3   Note: Show the main steps in separation of variables. To find the eigenvalues, eignefunctions and the constants, use Review, Identities, Formulas and Theorems. You may use Mathematica to find the Fourier coefficients.

2. See 2.5 #6
   Hints: State the main steps in separation of variables. Keep $-\gamma^2$ with the x terms. To find the eigenvalues, eignefunctions and the constants, use Review, Identities, Formulas and Theorems.

3. Find all nontrivial solutions of the boundary value problem $\phi''(x) = -\lambda\phi$, for $0 < x < a$, with boundary conditions $\phi'(0) = 0$ and $\phi(a) = 0$.
   Hints: Consider the cases $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$, in that order. Also, see Review, Identities, Formulas and Theorems.

4. Show that if $f(x) = \sum_{n=1}^{\infty} a_n \cos\frac{(2n-1)\pi x}{2a}$ for $0 < x < a$, then $a_n = \frac{2}{a}\int_0^a f(x)\cos\frac{(2n-1)\pi x}{2a}\,dx$.
   Hints: This was an exercise in Brief Notes. Use the hints stated there. See Review, Identities, Formulas and Theorems or your first homework.

5. Solve $\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, & 0 < x < a,\ t > 0 \\ \frac{\partial u}{\partial x}(0,\ t) = u(a,\ t) = 0, & t > 0 \\ u(x,\ 0) = f(x), & 0 < x < a \end{cases}$.
   Note: Show the main steps in separation of variables. Use the last two problems to find the eigenvalues, eignefunctions and the constants.

6. Find all nontrivial solutions of the boundary value problem $\phi''(x) = -\lambda\phi$, for $-a < x < a$, with boundary conditions $\phi(-a) = \phi(a)$ and $\phi'(-a) = \phi'(a)$.
   Hints: Consider the cases $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$, in that order. To solve the last case, consider the boundary conditions as a system of equations. Also, see Review, Identities, Formulas and Theorems.

7. Free points!

8. Free points!

9. Free points!

10. Free points!