Math 3710 HOMEWORK #7 Name:

Due 10/13/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. See 2.1 #5
   Hints: Consider the cases $u(0,\,t) > T(t)$ and $u(0,\,t) < T(t)$ and follow the derivation of equations 9 and 10. Also, see class notes.

2. See 2.2 #6

3. See 2.2 #7

4. See 2.2 #8
   Hint: To find the general solution of the 2nd order linear nonhomogeneous ODE, add the general solution of the homogeneous part to a particular solution. To find a particular solution, apply the method of undetermined coefficients. See Review, Identities, Formulas and Theorems.

5. Find all nontrivial solutions of the eigenvalue value problem $\phi''(x) = -\lambda\,\phi$, for $0 < x < a$, with boundary conditions $\phi(0) = 0$ and $\phi(a) = 0$.
   Hints: Consider the cases $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$, in that order. See class notes. Also, see Review, Identities, Formulas and Theorems.

6. See 2.3 #7 Note: Show all steps in separation of variables. But, you do not need to consider all three cases for λ. Just completely show the case that does not lead to a trivial solution. You may use Mathematica to find the Fourier coefficients.

7. Solve $\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, & 0 < x < a,\ t > 0 \\ u(0,\,t) = u(a,\,t) = 0, & t > 0 \\ u(x,\,0) = 6\sin\frac{9\pi x}{a}, & 0 < x < a \end{cases}$.
   Note: State the main steps in separation of variables. But, you do not need to show all details or the three cases for λ. Use Review, Identities, Formulas and Theorems. It is trivial to find the F. coefficients!

8. Find all nontrivial solutions of the eigenvalue value problem $\phi''(x) = -\lambda\,\phi$, for $0 < x < a$, with boundary conditions $\phi'(0) = 0$ and $\phi'(a) = 0$.
   Hints: Consider the cases $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$, in that order. See class notes. Also, see Review, Identities, Formulas and Theorems.

9. See 2.4 #1 Note: State the main steps in separation of variables. But, you do not need to consider all three cases for λ. Just completely show cases that do not lead to a trivial solution.You may use Mathematica to find the F. coefficients and to graph. Use $k = 1$, $a = 10$, $T_1 = 10$ and graph the steady-state solution, $u(x,\,0)$ and $u(x,\,1)$ on the same coordinate system.

10. See 2.4 #2 Note: State the main steps in separation of variables. But, you do not need to show all details or the three cases for λ. Use Review, Identities, Formulas and Theorems. You may use Mathematica to find the F. coefficients and to graph. Use $k = 1$, $a = 10$, $T_0 = 20$, $T_1 = 10$ and graph the steady-state solution, $u(x,\,0)$ and $u(x,\,1)$ on the same coordinate system.