Math 3710 HOMEWORK #6 Name:

Due 10/06/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.
For problems 1-6, let $f(x) = x^2$, $h(x) = \frac{1}{3}x^3 - \frac{\pi^2}{3}x$, and $k(x) = x^3$, all for $0 < x < \pi$. In an earlier homework we found the Fourier cosine series of f, and by applying the convergence theorem, we can show that $x^2 = \dfrac{\pi^2}{3} + \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2}\cos(nx)$ for $0 \le x \le \pi$. (Notice the equality holds at the endpoints. Do you know why?)

1. Show $x^2 = \displaystyle\sum_{n=1}^{\infty} \frac{2[(2-n^2\pi^2)(-1)^n - 2]}{n^3\,\pi}\sin(nx)$ for $0 \le x < \pi$, by finding the Fourier sine series of f and discussing its convergence. Pay attention to the endpoints! Note: You may use Mathematica or Review, Identities, Formulas and Theorems for integration.

2. Determine which of the above two Fourier series of f can be differentiated term-by-term and use the appropriate one to find a Fourier series for $f'(x) = 2x$, $0 < x < \pi$. Explain!
   Hints: Determine whether f satisfies the required hypotheses for term-by-term integration for the F. cosine series or F. sine series. Then apply the term-by-term differentiation to the appropriate series.

3. Use the result of the last problem to show that $2x = \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n}\sin(nx)$ for $0 \le x < \pi$ and $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} \cdots = \dfrac{\pi}{4}$.

   Hints: Solve the last problem correctly and apply the convergence theorem to the new F. series. Evaluate the equation at an appropriate x value. Note: Pay attention to the equality at $x = 0$.

4. Show $h(x) = \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^n}{n^3}\sin(nx)$ for $0 \le x \le \pi$, by term-by-term integration of the given Fourier cosine series of f.
   Note: Be sure to apply the convergence theorem to show the equality at the endpoints.

5. Use the result of problems 3 and 4 to find the Fourier sine series of $k(x)$.

6. Can we differentiate the Fourier sine series of $h(x)$ term-by-term? Explain! Can we differentiate the Fourier sine series of $k(x)$ term-by-term? What is the difference? Explain!

7. Free points!

8. Free points!

9. Free points!

10. Free points!