Math 3710 HOMEWORK #4 Name:

Due 9/22/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Find the Fourier sine series of $f(x) = \begin{cases} \frac{2x}{a}, & 0 < x \le \frac{a}{2} \\ 2 - \frac{2x}{a}, & \frac{a}{2} < x \le a \end{cases}$, where a is a positive constant.

   Hint: Use $\int_0^a = \int_0^{\frac{a}{2}} + \int_{\frac{a}{2}}^a$ and apply the integration by parts technique.

2. See 1.3 #2(a, b)

3. See 1.3 #2(c, d)

4. Graph the Fourier series of the function $f(x) = \begin{cases} -x-2 & \text{for } -2 \le x < -1 \\ x & \text{for } -1 \le x \le 1 \\ -x & \text{for } 1 < x \le 2 \end{cases}$ for the interval $-7 \le x \le 7$. Note: Do not find its Fourier coefficients!
   Hint: Use the convergence theorem of Fourier series.

5. Find and use the Fourier cosine series of $f(x) = x^2$, $0 < x < \pi$, to show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

   Hint: For integration, do integration by parts twice, or use Review, Identities, Formulas and Theorems or Mathematica. Note: There are calculus based proofs of this result!

6. See 1.3 #4
   Hints: Consider odd and even extensions of the function. Pay special attention to the end-points $x = 0$ and $x = a$.

7. See 1.3 #5

8. See 1.3 #6

9. Free points!

10. Free points!