Math 3710 HOMEWORK #2 Name:

Due 9/8/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

For the next three problems, consider the ODE $\phi''(x) = r^2\,\phi(x)$, where r is a positive constant. We know that $\phi_1(x) = e^{-rx}$ and $\phi_2(x) = e^{rx}$ are two linearly independent solutions of it and its general solution is $\phi(x) = c_1\,e^{-rx} + c_2\,e^{rx}$. Note: For the definition and properties of hyperbolic trigonometric functions, $\sinh x$ (pronounced "cinch of x") and $\cosh x$ (pronounced "kosh of x"), see Review, Identities, Formulas and Theorems.

1. Show that $\phi(x) = c_1\,\cosh(rx) + c_2\,\sinh(rx)$ is also a general solution of the above ODE.
2. Show that $\phi(x) = c_1\,\cosh r(x - x_0) + c_2\,\sinh r(x - x_0)$, where x_0 is a constant, is another general solution of the above ODE.
3. Show that $\phi(x) = c_1\,\sinh r(x - x_0) + c_2\,\sinh(rx)$, where x_0 is a nonzero constant, is yet another general solution of the above ODE.

   Hints for the above three problems: A general solution of a 2nd order linear homogeneous ODE is the linear combination of any pair of linearly independent solutions of it. Use the fact that a linear combination of solutions of a linear homogeneous ODE is also a solution of the same ODE to establish that each stated hyperbolic function is a solution. (You may also check them directly!) Then show each given pair of hyperbolic function solutions are linearly independent by verifying that their Wronskian is not zero. See Review, Identities, Formulas and Theorems.
4. Suppose f is an even differentiable function. Show that f' is an odd function.
   Hint: Since f is an even function, $f(-x) = f(x)$ for all x in the domain of f. Differentiate $f(-x) = f(x)$ using implicit differentiation.
5. Suppose f is an odd differentiable function. Show that f' is an even function.
   Hint: Since f is an odd function, $f(-x) = -f(x)$ for all x in the domain of f. Differentiate $f(-x) = -f(x)$ using implicit differentiation.
6. Suppose differentiable function f is periodic with period p. Show that f' is also a periodic function with period p.
   Hint: Since f is p-periodic, $f(x + p) = f(x)$ for all x in the domain of f. Differentiate $f(x + p) = f(x)$ using implicit differentiation.
7. Suppose piecewise continuous function g is p-periodic. Show that $\displaystyle\int_c^{c+p} g(x)\,dx = \int_0^p g(x)\,dx$ for any number c.
   Hints: $\int_c^{c+p} g(x)\,dx = \int_c^p g(x)\,dx + \int_p^{c+p} g(x)\,dx$. In the second integral, apply the u-substitution, $u = x - p$, and the periodicity property of g. Then, combine the two integrals.
8. Suppose piecewise continuous function g is p-periodic and $\displaystyle\int_0^p g(x)\,dx = 0$. Show that $G(x) = \displaystyle\int_0^x g(t)\,dt$ is also p-periodic.
   Hint: Write $G(x+p) = \int_0^{x+p} g(t)\,dt$ as the sum of two appropriate integrals and use the result of the last problem.
9. Free points!
10. Free points!