Math 3710 HOMEWORK #13 Name:

Due 12/04/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 5.3 # 6
   Hint: See the class notes.

2. Solve the following two-dimensional EVP, by converting it to two one-dimensional EVP's.

$$\begin{aligned}
&\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\lambda\, \phi(x,\, y)\,, \quad 0 < x < a\,,\ 0 < y < b \\
&\frac{\partial \phi}{\partial y}(x,\, 0) = \frac{\partial \phi}{\partial y}(x,\, b) = 0, \quad 0 < x < a \\
&\frac{\partial \phi}{\partial x}(0,\, y) = \frac{\partial \phi}{\partial x}(a,\, y) = 0, \quad 0 < y < b
\end{aligned}$$

   Note: Do not just quote the final answer from Review, Identities, Formulas and Theorems. Just use it for the solutions of one-dimensional EVP's. Hint: Let $\phi(x,\, y) = X(x)Y(y)$.

3. Sec 5.3 # 7(b)
   Note: The boundary conditions are $\frac{\partial u}{\partial x}(0,\, y,\, t) = \frac{\partial u}{\partial x}(a,\, y,\, t) = \frac{\partial u}{\partial y}(x,\, 0,\, t) = \frac{\partial u}{\partial y}(x,\, b,\, t) = 0$. Use the result of problem 2 or the Review, Identities, Formulas and Theorems for the solution of the two-dimensional EVP and the constants formulas. Find the constants!

4. Sec 5.4 # 5

5. Sec 5.4 # 6

6. Sec 5.5 # 1
   Hint: Start with the general solution stated in the book, right before this problem, and apply the boundary conditions and use the properties of Bessel functions.

7. Sec 5.6 # 3
   Hint: Add the condition: $v(0,\, t)$ bounded. See class notes. Calculate the constants using a u-substitution and the Bessel function property: $\int x\, J_0(x)\, dx = x\, J_1(x) + C$.

8. Solve

$$\begin{aligned}
&\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\,, && 0 < r < a\,,\ t > 0 \\
&u(a,\, t) = 0\,, && t > 0 \\
&u(0,\, t)\ \text{bounded}\,, && t > 0 \\
&u(r,\, 0) = f(r)\,, && 0 < r < a \\
&\frac{\partial u}{\partial t}(r,\, 0) = 0\,, && 0 < r < a.
\end{aligned}$$

   Hint: See class notes.

9. Free points!

10. Free points!