Math 3710 HOMEWORK #11 Name:

Due 11/10/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Solve

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, && 0 < x < a,\ t > 0\\
&\frac{\partial u}{\partial x}(0,\, t) = 0, && t > 0\\
&\frac{\partial u}{\partial x}(a,\, t) = 0, && t > 0\\
&u(x,\, 0) = f(x), && 0 < x < a\\
&\frac{\partial u}{\partial t}(x,\, 0) = 0, && 0 < x < a
\end{aligned}$$

   using the method of separation of variables.
   Hints: State the main steps in separation of variables and use Review, Identities, Formulas and Theorems to solve the EVP.

2. Solve the last problem using the d'Alembert's solution.
   Hint: Start with $u(x,\, t) = \phi(x + ct) + \psi(x - ct)$. Be sure to explain how the extension of f is made. Use the earlier results, such as the symmetry properties of a function and its derivative, as needed.

3. Show that the solutions of the last two problems are equivalent.
   Hint: Use F. series representation of $f(x)$ in the separation of variables method and plug it into the d'Alembert's solution and simplify.

4. Sec 3.4 #8
   Hints: Keep γ^2 with the T's. Use Review, Identities, Formulas and Theorems for solving the EVP. The observations the problems refer to are the three numbered statements in the page 238.

5. Show that, in polar coordinates, $\nabla^2 u = \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}$.
   Hint: See class notes.

6. Sec 4.1 #6

7. Sec 4.2 #5
   Note: You may use the Review, Identities, Formulas and Theorems handout. Level curves are of the form $u(x,\, y) =$ constant. You may use Mathematica.

8. Sec 4.2 #6

9. Free points!

10. Free points!