Math 3710 HOMEWORK #10 Name:

Due 11/03/2023, 8:30 a.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 3.1 #3
2. Sec 3.2 #4
   Hints: Use Review, Identities, Formulas and Theorems for solving the EVP. It is trivial to find the constants!
3. Sec 3.2 #12
   Hints: Use Review, Identities, Formulas and Theorems for solving the EVP. The solution will be of the form $u = \sum_{n=1}^{\infty} e^{-kt}[a_n \cos(\nu_n t) + b_n \sin(\nu_n t)] \sin \frac{n\pi x}{a}$, for some ν_n values. Find the constants a_n and b_n. (You may use Mathematica for integration.) Also, $\sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{a} = 0$ implies that $c_n = 0$ for $n = 1, 2, \cdots$. (Why?)
4. Sec 3.2 #13
   Hints: The solution will be of the form $u = \sum_{n-1}^{\infty} [a_n \cos(\nu_n ct) + b_n \sin(\nu_n ct)] \sin \frac{n\pi x}{a}$, for some ν_n values. See the hint in the last problem.
5. Sec 3.3 #3
6. Sec 3.3 #4
7. Suppose that $\phi(w)$ and $\psi(z)$ are twice differentiable functions. Show that $u(x, t) = \phi(x + ct) + \psi(x - ct)$ satisfies the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$.
8. (a). Show that if $g(x)$ is an even (or odd) function, then $G(x) = \int_0^x g(\xi)\, d\xi$ is an odd (or even function). (b). Show that if g is an **odd** p-periodic function, then $\int_0^p g(x)\, dx = 0$.
   Hints: $G(-x) = \int_0^{-x} g(\xi)\, d\xi$. Use substitution $u = -\xi$. In an earlier HW, we showed that if piecewise continuous function $g(x)$ is p-periodic, then $\int_c^{c+p} g(x)\, dx = \int_0^p g(x)\, dx$. Use $c = -p/2$.
   Remark: From this problem and an earlier HW, if piecewise continuous function $g(x)$ is p-periodic and $\int_0^p g(x)\, dx = 0$ (or g is an **odd** function), then $G(x) = \int_0^x g(\xi)\, d\xi$ is also a p-periodic function (and $G(x)$ is an **even** function).
9. Solve
   $$\begin{aligned} \frac{\partial^2 u}{\partial x^2} &= \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, && 0 < x < a,\ t > 0 \\ u(0, t) &= u(a, t) = 0, && t > 0 \\ u(x, 0) &= f(x), && 0 < x < a \\ \frac{\partial u}{\partial t}(x, 0) &= g(x), && 0 < x < a \end{aligned}$$
   using the d'Alembert's solution.
   Hint: Start with $u(x, t) = \phi(x + ct) + \psi(x - ct)$. Be sure to explain how the extensions of f and G or g are made. See the class notes.
10. Free points!